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NEW BOUNDS ON THE PERFORMANCE PARAMETERS OF A THERMOELECTRIC GENERATOR

Jincan Chen* and Bjarne Andresen**

Abstract

The influence of three main irreversibilities existing in thermoelectric devices on the performance of a thermoelectric generator is investigated by means of nonequilibrium thermodynamics and finite-time thermodynamics. The efficiency and the power output of the thermoelectric generator are optimized. The maximum efficiency and the maximum power output are determined. The power output versus efficiency curves illustrate clearly that the two operating points corresponding to maximum efficiency and maximum power output approach one another as the thermal conductances between the thermoelectric device and its external heat reservoirs decrease. Finally, the optimal problem relative to the load matching involved in the design of a practical thermoelectric generator is discussed in detail.

Key Words

Thermodynamics, thermoelectric generator, irreversibility, performance bound

1. Introduction

Since semiconductor materials were discovered, the practical exploitation of thermoelectric devices has become very attractive. Thermoelectric generators have been developed for many uses ranging from small flame-operated power supplies for automatic control systems to several kilowatt-size thermopiles for prototype evaluation of marine propulsion systems. A large number of authors have analyzed the performance of a thermoelectric generator by means of nonequilibrium thermodynamics [1–6]. In recent years, several authors have studied the influence of finite-rate heat transfer between the thermoelectric device and its external heat reservoirs on the performance of the thermoelectric generator [7–10] and obtained some new results.

In the present paper we will investigate the general characteristics of a thermoelectric generator when the thermal conductances between the thermoelectric device and its external heat reservoirs are different. New bounds on the key performance parameters of a thermoelectric generator will be given.

2. Three Main Irreversible Losses

A thermoelectric generator which converts part of a quantity of heat absorbed directly into d.c. electricity is composed primarily of p-type and n-type semiconductor elements. Figure 1 presents a simplified view of a thermoelectric generation system. Here q_h is the net rate of heat input from the heat source at temperature T_h to the thermoelectric device and q_c the net rate of heat rejection from the thermoelectric device to the heat sink at temperature T_c. The power output p of the thermoelectric generator is received by the load resistance R_L. The term I represents the electrical current. The terms T_1 and T_2 are the temperatures of the two junctions in the thermoelectric device. The term q_k is the heat leak from the hot junction at temperature T_1 to the cold junction at temperature T_2 via the thermoelectric elements. The terms k_h and k_c are the thermal conductances between the hot and the cold junctions of the thermoelectric device and their respective heat reservoirs. The term k is the thermal conductance of the thermoelectric device. The thermoelectric device is assumed to be insulated, both electrically and thermally, from its surroundings except at the junction-reservoir contacts [7, 9].

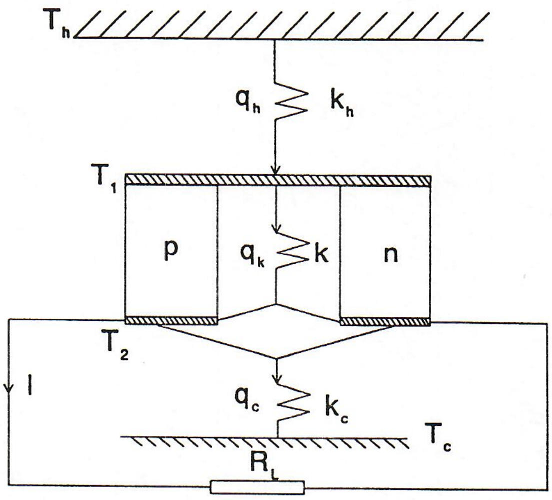

Figure 1. Schematic diagram of a thermoelectric generator operating between reservoirs at T_h and T_c.

The operation of a thermoelectric generator is based on the Seebeck effect, first observed by Seebeck in 1821. When the thermoelectric device works as a generator, the heat fluxes

$$q_1 = \alpha I T_1 \tag{1}$$

and

$$q_2 = \alpha I T_2 \tag{2}$$

* Department of Physics, Xiamen University, Xiamen 361005, People's Republic of China

** Ørsted Laboratory, Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

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are absorbed by the hot junction and released from the cold junction, respectively, due to the Seebeck effect; α is the seebeck coefficient. At the same time, there are two additional effects in play and always present in thermoelectric devices: Joule heating due to the electrical current and heat leak between the two junctions.

The traditional treatment ignored the heat resistances between the thermoelectric device and its external heat reservoirs and assumed that the thermal conductances between the thermoelectric device and its external heat reservoirs are infinitely large. This implies that the heat transfer areas of the two junctions in the device are infinitely large, in engineering terms a most unrealistic situation. In order to make the results approximate the performance of a practical thermoelectric generator more closely, the influence of the irreversibility of finite-rate heat transfer between the thermoelectric device and its external heat reservoirs is considered in this paper.

It is often assumed that heat transfer obeys a Newtonian law [11, 12]. Then q_h, q_c, and q_k may be expressed as

$$q_h = k_h(T_h - T_1) \tag{3}$$

$$q_c = k_c(T_2 - T_c) \tag{4}$$

and

$$q_k = k(T_1 - T_2) \tag{5}$$

where $k = k_pA_p/L_p + k_nA_n/L_n$. The parameters k_p and k_n are the thermal conductivities, A_p and A_n are the cross-section areas, and L_p and L_n are the lengths of p-type and n-type semiconductor elements. It is assumed that these parameters are constants.

According to the above analysis, three main losses [13] exist in the thermoelectric generation system: Joulean heat production inside the thermoelectric device, heat leak through the thermoelectric device, and irreversible finite-rate heat transfer between the thermoelectric device and its external heat reservoirs. We can further prove [14] that the Joulean heat rate

$$q_j = RI^2 \tag{6}$$

inside the thermoelectric elements flows equally to the hot and the cold junctions of the device for any values of the thermal conductances k_h and k_c; $R = L_p/(\sigma_pA_p) + L_n/(\sigma_nA_n)$ is the total electrical resistance of the device and the electrical conductivities σ_p and σ_n of p-type and n-type semiconductor elements are assumed to be constants. Thus one has

$$q_h = q_1 + q_k - q_j/2 = \alpha IT_1 + k(T_1 - T_2) - RI^2/2 \tag{7}$$

and

$$q_c = q_2 + q_k + q_j/2 = \alpha IT_2 + k(T_1 - T_2) + RI^2/2 \tag{8}$$

Note that the constant b from [8] should be chosen equal to 1/2 for any values of the thermal conductances k_h and k_c.

3. The Efficiency and Power Output

Solving equations (3), (4), (7), and (8), we obtain the junction temperatures

$$T_1 = \frac{k(k_hT_h + k_cT_c + RI^2) + (k_c - \alpha I)(k_hT_h + RI^2/2)}{(k_h + \alpha I)(k_c - \alpha I) + k(k_h + k_c)} \tag{9}$$

and

$$T_2 = \frac{k(k_hT_h + k_cT_c + RI^2) + (k_h + \alpha I)(k_cT_c + RI^2/2)}{(k_h + \alpha I)(k_c - \alpha I) + k(k_h + k_c)} \tag{10}$$

Substituting (9) and (10) into (3) and (4), one has

$$q_h = k_h\frac{k[k_c(T_h - T_c) - RI^2] + (k_c - \alpha I)(\alpha IT_h - RI^2/2)}{(k_h + \alpha I)(k_c - \alpha I) + k(k_h + k_c)} \tag{11}$$

and

$$q_c = k_c\frac{k[k_h(T_h - T_c) + RI^2] + (k_h + \alpha I)(\alpha IT_c + RI^2/2)}{(k_h + \alpha I)(k_c - \alpha I) + k(k_h + k_c)} \tag{12}$$

From the usual definitions of the efficiency η and the power output p of a thermoelectric generator we obtain

$$\eta = 1 - q_c/q_h$$
$$= 1 - \frac{kk_c[k_h(T_h - T_c) + RI^2] + k_c(k_h + \alpha I)(\alpha IT_c + RI^2/2)}{kk_h[k_c(T_h - T_c) - RI^2] + k_h(k_c - \alpha I)(\alpha IT_h - RI^2/2)} \tag{13}$$

and

$$p = q_h - q_c$$
$$= I\frac{\alpha k_hk_c(T_h - T_c) - [\alpha^2(k_hT_h + k_cT_c) + Rk(k_h + k_c) + Rk_hk_c]I + \alpha R(k_h - k_c)I^2/2}{(k_h + \alpha I)(k_c - \alpha I) + k(k_h + k_c)} \tag{14}$$

For convenience let $i = RI/\alpha$. Then (11)-(14) may be rewritten as

$$q_h = k\frac{T_h - T_c + ZT_hi - (1/2 + B_2 + B_2ZT_h)Zi^2 + (1/2)B_2Z^2i^3}{B_1 + B_2 + (1 + B_1Zi)(1 - B_2Zi)} \tag{15}$$

$$q_c = k\frac{T_h - T_c + ZT_ci + (1/2 + B_1 + B_1ZT_c)Zi^2 + (1/2)B_1Z^2i^3}{B_1 + B_2 + (1 + B_1Zi)(1 - B_2Zi)} \tag{16}$$

$$\eta = 1 - \frac{T_h - T_c + ZT_ci + (1/2 + B_1 + B_1ZT_c)Zi^2 + (1/2)B_1Z^2i^3}{T_h - T_c + ZT_hi - (1/2 + B_2 + B_2ZT_h)Zi^2 + (1/2)B_2Z^2i^3} \tag{17}$$

$$p = k\frac{Z(T_h - T_c)i - [1 + B_1 + B_2 + Z(B_2T_h + B_1T_c)]Zi^2 + (1/2)(B_2 - B_1)Z^2i^3}{B_1 + B_2 + (1 + B_1Zi)(1 - B_2Zi)} \tag{18}$$

where $B_1 = k/k_h$, $B_2 = k/k_c$, and where $Z = \alpha^2/(kR)$ is referred to as the figure of merit of the elements [2, 7, 10].

From (15)–(18), we can easily generate Figures 2–4. Figure 2 shows that the rates of heat input q_h and heat rejection q_c are monotonically increasing functions of the reduced current i. Figures 3 and 4 show that there exists a maximum efficiency and a maximum power output for a thermoelectric generator. It is seen clearly from Figures 3 and 4 that the maximum efficiency $\eta_{\max}$ and the maximum power output $p_{\max}$ of a thermoelectric generator decrease as the thermal conductances k_h and k_c decrease.

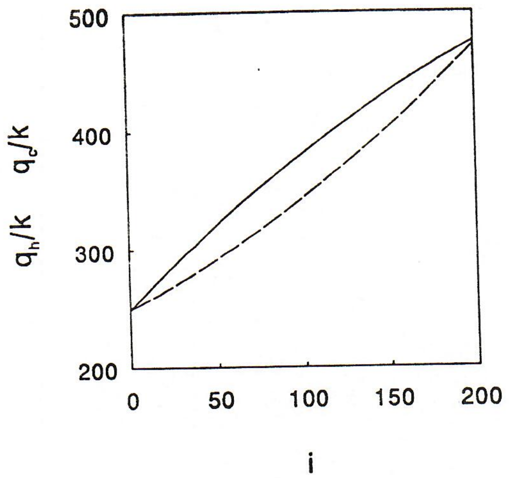

Figure 2. The rates of heat input q_h (solid line) and heat rejection q_c (dashed line) versus the reduced current i. Plots are presented for $B_1 = B_2 = 0.1$, $Z = 0.003K^{-1}$, $T_h = 600K$, and $T_c = 300K$.

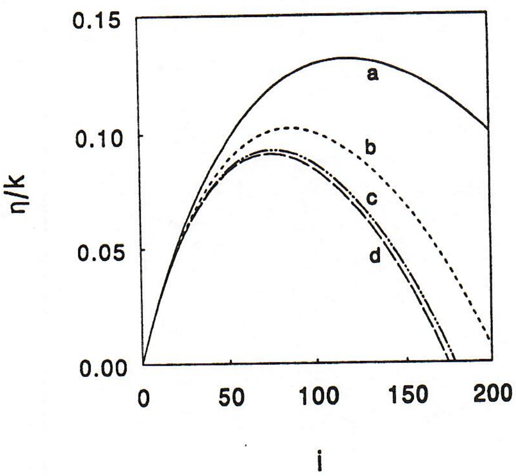

Figure 3. The efficiency η versus the reduced current i. The values of Z, T_h, and T_c are the same as those used in Figure 2. Curves a, b, c, and d correspond to the cases of $B_1 = B_2 = 0$, $B_1 = B_2 = 0.1$, $B_1 = 0.2$ and $B_2 = 0.1$, and $B_1 = 0.1$ and $B_2 = 0.2$, respectively.

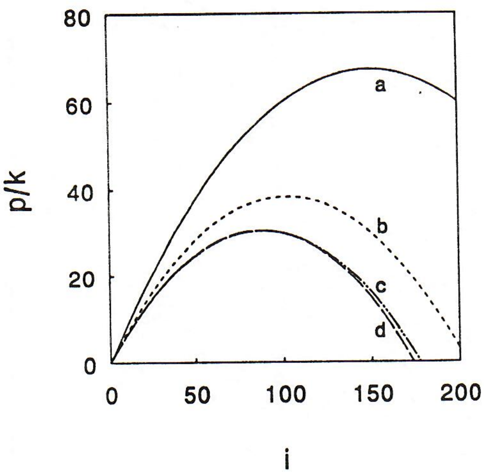

Figure 4. The power output p versus the reduced current i. The values of Z, T_h, T_c, B_1, and B_2 are the same as those used in Figure 3.

4. Maximum Efficiency

Using Equation (17) and the extremum condition

$$\partial\eta/\partial i = 0 \tag{19}$$

we find that when the thermoelectric generator attains its maximum efficiency, the corresponding reduced current $i_{\eta m}$ is determined by

$$\begin{aligned}-(1/2)(B_1C_2 + B_2C_1)Z^2 i_{\eta m}^4 + (B_1T_h - B_2T_c)Z^3 i_{\eta m}^3 \\ +[(3/2)(B_1 - B_2)(T_h - T_c)Z^2 + (C_1T_h + C_2T_c)Z]i_{\eta m}^2 \\ +2(C_1 + C_2)(T_h - T_c)i_{\eta m} - (T_h - T_c)^2 Z = 0\end{aligned} \tag{20}$$

where $C_1 = (1/2 + B_1 + B_1ZT_c)Z$ and $C_2 = (1/2 + B_2 + B_2ZT_h)Z$. Equation (20) shows that for given parameters B_1, B_2, Z, T_h, and T_c, there is an analytic solution for $i_{\eta m}$, albeit a complicated one. When the two thermal conductances to the surroundings are identical, $B_1 = B_2 = B$, Equation (20) may be reduced to

$$\begin{aligned}-(1/2)BCZ^3 i_{\eta m}^4 + B(T_h - T_c)Z^3 i_{\eta m}^3 \\ +[(1/2+B)(T_h+T_c)+2BZT_hT_c]Z^2 i_{\eta m}^2 + 2C(T_h-T_c)Z i_{\eta m} - (T_h-T_c)^2 Z = 0\end{aligned} \tag{21}$$

where $C = 1 + 2B + BZ(T_h + T_c)$.

When $B = 0$, corresponding to infinitely better conductance to the reservoirs than across the semiconductors, we obtain an analytic solution,

$$i_{\eta m} = \frac{2}{Z}\frac{T_h - T_c}{T_h + T_c}[\sqrt{1 + (1/2)Z(T_h + T_c)} - 1] \tag{22}$$

from (21). Substituting (22) into (17) and (18), we obtain the maximum efficiency

$$\eta_{\max} = (1 - \frac{T_c}{T_h})\frac{\sqrt{1 + (1/2)Z(T_h + T_c)} - 1}{\sqrt{1 + (1/2)Z(T_h + T_c)} + T_c/T_h} = \eta_0 \tag{23}$$

with the corresponding power output

$$p_m = 2k\frac{(T_h - T_c)^2}{T_h + T_c}\{[1 + \frac{4}{Z(T_h + T_c)}][\sqrt{1 + (1/2)Z(T_h + T_c)} - 1] - 1\} \tag{24}$$

Equation (23) shows that the maximum efficiency of a thermoelectric generator is always smaller than the efficiency of a reversible Carnot heat engine operating across the same temperature range. The origin is the thermal conductance k of the semiconductor elements, which is always larger than zero so that the heat leak loss is unavoidable.

5. Maximum power output

From (18) and the extremum condition

$$\partial p/\partial i = 0 \tag{25}$$

we obtain the reduced current i_{pm} at maximum power output

$$\begin{aligned}
&-(1/2)(B_2 - B_1)B_1B_2Z^4 i_{pm}^4 - (B_1 - B_2)^2 Z^3 i_{pm}^3 \\
&+\{B_1B_2(T_h - T_c)Z^3 + (3/2)(1+B_1+B_2)(B_2-B_1)Z^2 \\
&-(B_1 - B_2)[1 + B_1 + B_2 + Z(B_2T_h + B_1T_c)]Z^2\} i_{pm}^2 \\
&-2(1+B_1+B_2)[1+B_1+B_2+Z(B_2T_h+B_1T_c)]Z i_{pm} + (1+B_1+B_2)(T_h-T_c)Z = 0
\end{aligned} \tag{26}$$

Like $i_{\eta m}$, there is an analytic solution for i_{pm}, but again it is complicated. However, in the symmetric case, $B_1 = B_2 = B$ the analytic solution

$$i_{pm} = \frac{(1+2B)C}{B^2Z^2(T_h - T_c)}\left[1 - \sqrt{1 - \frac{B^2Z^2(T_h - T_c)^2}{(1+2B)C^2}}\right] \tag{27}$$

may be derived from (26). Substituting (27) into (18) and (17), we obtain the maximum power output

$$p_{\max} = k\frac{C}{2B^2Z}\left[1 - \sqrt{1 - \frac{B^2Z^2(T_h - T_c)^2}{(1+2B)C^2}}\right] \tag{28}$$

with the corresponding efficiency

$$\eta_m = \frac{T_h - T_c - Ci_{pm}}{(1/2)BZi_{pm}^2 - (1/2 + B + BZT_h)i_{pm} + T_h + (T_h - T_c)/(Zi_{pm})} \tag{29}$$

In the large-conductance limit $B = 0$, i_{pm}, $p_{\max}$, and η_m may be expressed simply as

$$i_{pm} = (T_h - T_c)/2 \tag{30}$$

$$p_{\max} = \alpha^2(T_h - T_c)^2/(4R) \equiv p_0 \tag{31}$$

and

$$\eta_m = \frac{1}{2}\frac{1 - T_c/T_h}{1 - (1 - T_c/T_h)/4 + 2/(ZT_h)} \tag{32}$$

respectively. It is important to note that when $B = 0$, the electrical current at maximum power output

$$I_{pm} = \alpha i_{pm}/R = \alpha(T_h - T_c)/(2R) \tag{33}$$

is independent of k. When k_h and k_c are finite, the electrical current I_{pm} at maximum power output is dependent on k as well as k_h and k_c.

6. The Power Output Versus Efficiency Characteristics

From Equations (17) and (18), we can obtain the power output versus efficiency curves, as shown in Figure 5. It is seen once again that the influence of the irreversibility of finite-rate heat transfer between the device and its external heat reservoirs on the performance of a thermoelectric generator is substantial. Both the maximum power output and the maximum efficiency decrease as the thermal conductances k_h and k_c decrease.

Figure 5 illustrates clearly that maximum efficiency and maximum power output are two different operating points. They get closer to one another when the thermal conductances between the thermoelectric device and its external heat reservoirs decrease.

Figure 5 also shows that when a thermoelectric generator is operated in these parts of the $p - \eta$ curves with positive slope, the power output decreases as the efficiency decreases. These regions are not the optimal operating regions. The optimal operating region of a thermoelectric generator is situated in the parts of the $p - \eta$ curves with negative slope. Then the power output will increase as the efficiency decreases, and vice versa. Thus, the reduced

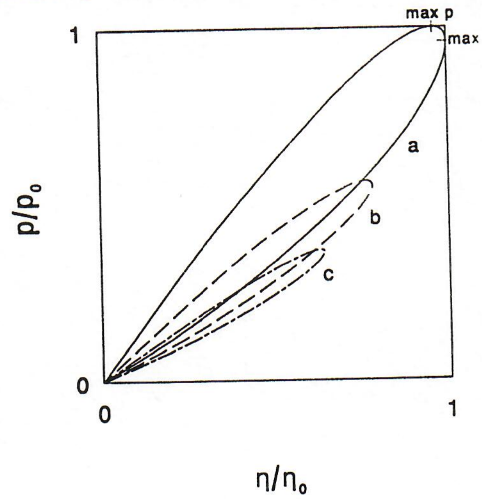

Figure 5. The power output ratio p/p_0 versus the efficiency ratio η/η_0. The values of Z, T_h, and T_c are the same as those used in Figure 2. Curves a, b, and c correspond to the cases of $B_1 = B_2 = 0$, 0.1, and 0.2, respectively.

current should be constrained by

$$i_{\eta m} \le i \le i_{pm} \tag{34}$$

such that the power output

$$p \ge p_m \tag{35}$$

and the efficiency

$$\eta \ge \eta_m \tag{36}$$

In the operation of a thermoelectric generator, the magnitude of the electrical current I is controlled directly by the load resistance. Consequently we must consider the optimal matching of the load resistance.

7. Optimal Load Matching

From (18) and the relation between the power output p and the load resistance R_L

$$p = I^2R_L \tag{37}$$

we obtain

$$R_L = R\frac{(T_h - T_c)/i - [1 + B_1 + B_2 + (B_2T_h + B_1T_c)Z] + (1/2)(B_2 - B_1)Zi}{B_1 + B_2 + (1 + B_1Zi)(1 - B_2Zi)} \quad (38)$$

Substituting i_{pm} and $i_{\eta m}$ into (38), we may get R_{Lp} and $R_{L\eta}$, respectively. According to (34), R_L must be constrained between R_{Lp} and $R_{L\eta}$, i.e.,

$$R_{Lp} \leq R_L \leq R_{L\eta} \quad (39)$$

Equation (39) is the condition of optimal load matching for a thermoelectric generator.

From (38), (17), and (18) we can plot the efficiency η versus load resistance R_L curve and the power output p versus load resistance R_L curve, as shown in Figure 6.

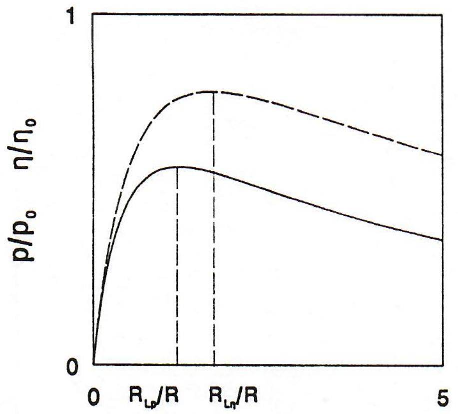

Figure 6. The power output ratio p/p_0 (solid line) and the efficiency ratio η/η_0 (dashed line) versus the ratio of the load resistance to internal resistance R_L/R. The values of Z, T_h, T_c, B_1, and B_2 are the same as those used in Figure 2.

It is well known that for a general d.c. power source, when the external resistance is equal to the internal resistance, the power output of the power source will attain a maximum. However, when the load resistance R_L is equal to the internal resistance R of a thermoelectric generator, the thermoelectric generator does not, in general, operate at maximum power output. For example, when $B_1 = B_2$, the matching condition of the load at maximum power output [10]

$$R_{Lp} = R\frac{C}{2(1+2B)}[1 + \sqrt{1 - \frac{2Z^2(T_h - T_c)^2}{(1+2B)C^2}}] \quad (40)$$

may be derived from (38) and (27). When the thermal conductances k_h and k_c are finite,

$$R_{Lp} > R \quad (41)$$

Only if $B_1 = B_2 = 0$, can we obtain the matching condition of $R_{Lp} = R$. The physical meaning of this new conclusion has been expounded in [10].

8. Conclusion

We have analyzed the general characteristics of a thermoelectric generator. The new performance bounds of the key parameters such as the power output and efficiency are determined. The matching condition of the load resistance derived here will be useful for the determination of the optimal operation of a practical thermoelectric generator.

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References

[1] J. Kaye & J.A. Welsh, *Direct conversion of heat to electricity* (New York: John Wiley & Sons, 1960).

[2] J.P. Holman, *Thermodynamics*, 3rd ed. (New York: McGraw-Hill Book Co., 1980).

[3] S.W. Angist, *Direct energy conversion*, 4th ed. (Boston, MA: Allyn & Bacon, 1982).

[4] B.D. Wood, *Applications of thermodynamics* (Reading, MA: Addison-Wesley, 1982).

[5] H.B. Callen, *Thermodynamics and an introduction to thermostatistics*, 2nd ed. (New York: John Wiley & Sons, 1985).

[6] A. Bejan, *Advanced engineering thermodynamics* (New York: John Wiley & Sons, 1988).

[7] J.M. Gordon, Generalized power versus efficiency characteristics of heat engines: The thermoelectric generator as an instructive illustration, *Am. J. Phys.*, *59*, 1995, 551–555.

[8] Z. Yan & J. Chen, Comment on "Generalized power versus efficiency characteristics of heat engines: The thermoelectric generator as an instructive illustration" by J.M. Gordon (Am. J. Phys. 59, 1991, 551-555), *Am. J. Phys.*, *61*, 1993, 380.

[9] C. Wu, Heat transfer effect on the specific power availability of heat engines, *Energy Convers. Mgmt.*, *34*, 1993, 1239–1247.

[10] Z. Yan & J. Chen, The load matching of a thermoelectric device at maximum power output, *13th Int. Conf. on Thermoelectrics*, USA, August 1994.

[11] B. Andresen, *Finite-time thermodynamics* (Copenhagen: Physics Laboratory II, University of Copenhagen, 1983).

[12] B. Andresen, P. Salamon, & R. S. Berry, Thermodynamics in finite time, *Phys. Today*, *37*, 1984, 62–70.

[13] M. Huleihil & B. Andresen, Generalized performance characteristics of heat pumps, *J. Appl. Phys.*, forthcoming.

[14] B. Andresen & J. Chen, unpublished.

Biography

Jincan Chen was born in 1954. He has been teaching at Xiamen University since 1980. Professor Jincan Chen has authored more than a hundred articles.