

Optimal heating and cooling strategies for heat exchanger design

Bjarne Andresen

Physics Laboratory, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

J. M. Gordon

Center for Energy and Environmental Physics, Jacob Blaustein Institute for Desert Research, Ben-Gurion University of the Negev, Sede Boqer Campus, Israel, and The Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beersheva, Israel

(Received 13 May 1991; accepted for publication 24 September 1991)

The optimal heating or cooling strategy that minimizes entropy production is derived for a simple class of common heat transfer processes that are constrained to proceed in a fixed, finite time. The empirical wisdom embodied in conventional single-pass counterflow heat exchanger design is examined in light of this solution. For judiciously selected system parameters, the counterflow heat exchanger *can* yield the optimal solution.

I. INTRODUCTION

In heat exchanger design the heat transfer is usually fixed, whereas the amount of entropy produced depends on the way in which the process is carried out (primarily the temperature gradient). Since the entropy produced is equivalent to availability lost, this means that an improved heat exchanger design which reduces the entropy production can in principle provide additional useful work somewhere else in the plant.

The following extensive energy consumers are often equipped to exploit this available work: factories with cogeneration capabilities, distillation plants driven by heat pumps or with reuse of the process heat at lower temperatures, and the electric utility providing the energy, especially if it also delivers district heating. For these installations minimizing entropy production can be a desirable objective.

We consider one particular type of problem toward illustrating the optimal heating/cooling strategy that minimizes entropy production. The process is constrained to proceed in a fixed, given time (as is common in industry where fixed production rates are a constraint). The problem selected is illustrative in that it has a simple closed-form analytic solution.

It is natural to ask how close conventional designs for heating and cooling processes are to the optimal one. We select the specific case of standard single-pass heat exchanger design in which three possibilities are usually considered: (1) parallel flow; (2) condensing flow (constant temperature for the effective heat reservoir); and (3) counterflow (see Fig. 1). Counterflow design is the usual choice primarily because its effectiveness (ratio of actual heat exchange rate to maximum possible heat exchange rate¹) is highest, and second because its lower-temperature gradients produce less entropy.

We show that the counterflow heat exchanger *can* represent the optimal solution for a somewhat idealized model which captures the essential physics of heat exchanger operation. Furthermore, common designs are often rather close to the optimal heating/cooling strategy. Empirically

the counterflow design has evolved as superior. Now one can appreciate just how beneficial it is in view of the optimal solution.

II. OPTIMAL SOLUTION FOR A SIMPLE HEATING/COOLING PROBLEM

For a system to be heated or cooled, the process must be completed in a given, fixed time τ . The system has a known heat capacity C and a time-dependent temperature $T(t)$ with required initial and final temperatures $T(0)$ and $T(\tau)$, respectively. Heating or cooling is effected from an external reservoir of known heat capacity C_0 and adjustable time-dependent temperature $T_0(t)$ (see Fig. 2). For simplicity of analysis, it is assumed that the only non-negligible thermal resistance is at the heat-transfer interface between system and reservoir, where there is a thermal conductance κ . In other words, heat equilibration within the system and the reservoir proceeds much faster than exchange between them (well-mixed media).

The objective is to find the heating/cooling strategy that minimizes entropy production when a given quantity of heat must be transferred between reservoir and system during a given time τ . The practical control variable is the reservoir temperature $T_0(t)$. In the analysis that follows, we consider the heating problem, wherein $T_0 > T$. The solution for the corresponding cooling problem, however, involves a simple change of sign.

The rate of entropy production dS_u/dt is

$$\frac{dS_u}{dt} = \kappa(T_0 - T) \left(\frac{1}{T} - \frac{1}{T_0} \right) \quad (1)$$

(which is always non-negative, as required by the second law of thermodynamics). The dynamic constraint for heat transfer rate q into the system is

$$q = \kappa(T_0 - T) = C \frac{dT}{dt}. \quad (2)$$

Two methods can be used to determine the optimal strategy. In the first more formal and cumbersome one, one

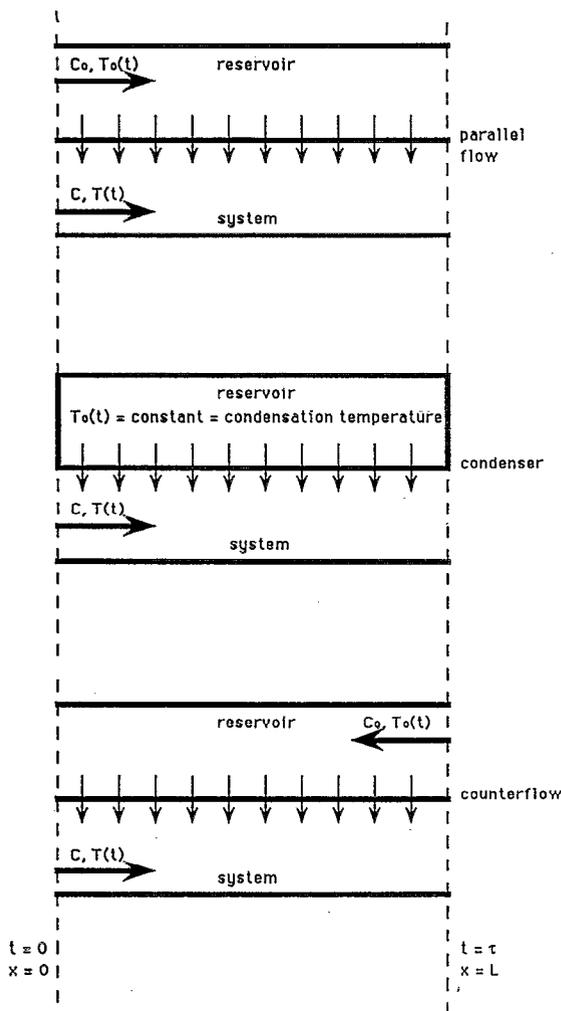


FIG. 1. Schematic of three single-pass heat exchanger designs (one-dimensional model): parallel flow, condenser, and counterflow. Horizontal arrows denote mass flow. Vertical arrows denote heat flow for heating of system by reservoir.

defines a modified Lagrangian L with a time-dependent Lagrange multiplier $\lambda(t)$:

$$L = \kappa(T_0 - T) \left(\frac{1}{T} - \frac{1}{T_0} \right) - \lambda \left(C \frac{dT}{dt} - \kappa(T_0 - T) \right). \quad (3)$$

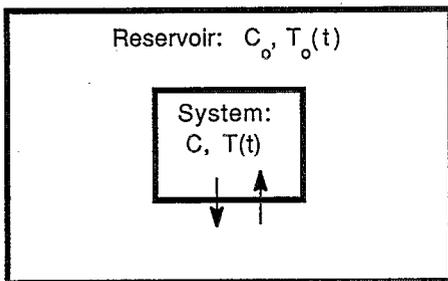


FIG. 2. Schematic of one-node thermal model for system exchanging heat with a variable-temperature (controllable) reservoir. Arrows denote heat exchange for heating or cooling.

The independent variables are T , dT/dt , and T_0 (and in principle dT_0/dt which, however, does not appear in the Lagrangian). The Euler-Lagrange equations to determine the optimal strategy then become

$$\frac{\partial L}{\partial T} - \frac{d}{dt} \frac{\partial L}{\partial (dT/dt)} = 0,$$

$$\frac{\partial L}{\partial T_0} = 0. \quad (4)$$

After some tedious algebra (not forgetting the time dependence of λ), one obtains the solution

$$T_0(t) = \beta T(t), \quad (5)$$

where β is a constant. Equation (2) then yields the final solution

$$T(t) = T(0) \exp\left(\frac{\kappa(\beta - 1)t}{C}\right), \quad (6)$$

where the constant β is determined from the condition that $T(0)$ and $T(\tau)$ are known:

$$\beta = 1 + \frac{C}{\kappa\tau} \ln\left(\frac{T(\tau)}{T(0)}\right). \quad (7)$$

The reservoir temperature, which is within our control, is then implemented according to Eqs. (5)–(7). The entropy production using this optimal strategy S_u^{opt} is obtained from integrating Eq. (1):

$$S_u^{\text{opt}} = \frac{\kappa(\beta - 1)^2 \tau}{\beta}. \quad (8)$$

The reversible solution of $T_0(t) = T(t)$, i.e., $\beta = 1$, yields zero entropy production but cannot satisfy the constraint that the process be completed in a fixed given time τ .

The second method of solution exploits the elegant proof of Ref. 2 that for any linear system the strategy for minimizing total entropy production corresponds to a constant rate of entropy production. Thus equating Eq. (1) to a constant immediately and simply yields Eq. (5). Using the constraint of Eq. (2), one again emerges with the rest of the optimal solution, Eqs. (6) and (7).

III. HEAT EXCHANGER DESIGN AND MINIMAL ENTROPY PRODUCTION STRATEGIES

One could now ask to what degree heating and cooling strategies that are used in factory and power plant settings mimic the strategy that minimizes entropy production. This issue, as well as the sensitivity of the optimal solutions to the functional form of the rate of heat transfer (e.g., radiative rather than linear), will be addressed in a separate publication.³ Here we restrict our attention to the special case of single-pass heat exchanger design as these are common elements in industrial and power plant energy systems and have functionally transparent results.

We consider finite temperature gradients as the sole source of irreversibility in the heat exchanger. In addition temperature gradients are modeled as one-dimensional only. These are, of course, idealizations since frictional

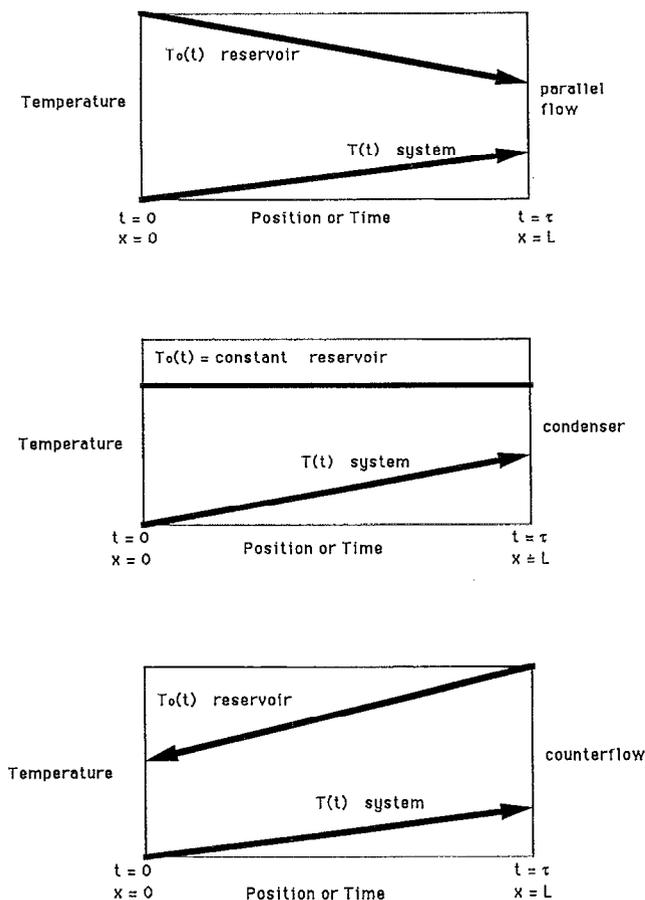


FIG. 3. Temperature vs time or, equivalently, heat exchanger position for parallel flow, condenser, and counterflow (schematic only). Arrows denote direction of mass flow.

flow losses exist, and fluid mixing is never perfect. However, for common well-designed heat exchangers these approximations capture the essential physics of the problem.¹ Given the added advantage of the functional simplicity of the corresponding results, we adopt the simple model of Sec. II for modeling a one-dimensional single-pass heat exchanger. As in Sec. II, for specificity of presentation, we analyze the case of the reservoir heating the system [$T_0(t) > T(t)$].

At fixed mass flow rates m and m_0 (for the system and environment, respectively), the time t and position x along the heat exchanger measured from the system inlet are equivalent variables (see Fig. 3). Consequently specifying the total process time τ or the length L of the heat exchanger amounts to the same. However, care must be exercised in transforming the results of Sec. II for the stationary system, Fig. 2, to the flow systems of Fig. 1 because the optimality condition (5) must be satisfied at each position along the heat exchanger:

$$T_0(x) = \beta T(x), \quad (9)$$

where

$$\beta = 1 + \frac{m\tilde{C}}{\tilde{\kappa}L} \ln \frac{T(L)}{T(0)} \quad (10)$$

is now given in terms of the mass flow rate m and the specific heat of the system \tilde{C} , thermal conductance per unit length of heat exchanger $\tilde{\kappa}$, and its total length L . The overhead symbol “ \sim ” denotes quantities normalized per unit mass or per unit length as required by the formulation of the flow system.

Similarly energy conservation at the interface requires that the rate of heat gained \tilde{q} by the system equals the rate of heat lost by the reservoir at each point:

$$\tilde{q} = \tilde{\kappa}(T_0 - T) = m\tilde{C} \frac{dT}{dx} = -m_0\tilde{C}_0 \frac{dT_0}{dx}, \quad (11)$$

which links $T(x)$ and $T_0(x)$ as

$$T_0(x) = -T(x) \frac{m\tilde{C}}{m_0\tilde{C}_0} + \alpha, \quad (12)$$

with $\alpha = \text{const}$.

The condenser heat exchanger with $T_0 = \text{const}$ is clearly incompatible with the optimality condition (5) or (9) since T is not constant. The other two types of heat exchangers may be analyzed together since counterflow is simply represented by $m_0 < 0$. Then energy conservation, Eq. (12), and optimality, Eq. (10), can be satisfied at the same time provided

$$\alpha = 0 \quad (13)$$

and

$$\beta = -\frac{m\tilde{C}}{m_0\tilde{C}_0}. \quad (14)$$

The material quantities, \tilde{C} , \tilde{C}_0 , $\tilde{\kappa}$, L , and the process variables $T(0)$, $T(L)$, m are usually given by design. The only remaining control variable is the reservoir mass flow m_0 , which must be adjusted to the value m_0^{opt} in order to ensure optimal operation of the process

$$m_0^{\text{opt}} = -\frac{1}{\tilde{C}_0} \left(\frac{1}{m\tilde{C}} + \frac{1}{\tilde{\kappa}L} \ln \frac{T(L)}{T(0)} \right)^{-1}. \quad (15)$$

Since the term in large parentheses is always positive, only a particular negative value of m_0 will produce minimum entropy. Thus parallel heat exchangers are never optimal. Counterflow heat exchangers are only optimal for a particular set of parameters. Note that the common case of “balanced” flow $m\tilde{C} = m_0\tilde{C}_0$ cannot be optimal.

IV. SUMMARY

There is an empirical wisdom that has evolved for equipment used in heat transfer and heat production. We have considered here a simple model for a specific but common heat transfer process in which the total heat transfer is fixed, as is the time for the process. Different heating and cooling strategies, however, result in markedly varying degrees of entropy production. The exercise of finding the heating or cooling strategy that minimizes entropy production is often of value to electric utilities and to factories with cogeneration installations.

The one-node thermal system in which a reservoir, the temperature of which can be controlled, heats a system via

a given thermal resistance in a given fixed time, yields a relatively simple optimal solution: reservoir and system temperatures are proportional to one another, and their time dependence is a simple exponential.

This simple one-node model also happens to capture the essential physics of well-designed single-pass heat exchangers.¹ Of the three common design options (parallel flow, condenser, and counterflow) only counterflow *can* be optimal, under judiciously chosen conditions. After solving for the generally optimal heating/cooling strategy for the one-node thermal model, one can determine the precise conditions for which counterflow heat exchangers represent the optimal heating/cooling strategy.

Design conditions are not always optimal, but common design parameters yield profiles that are close to optimal. For example, consider the common matched $m\tilde{C}=m_0\tilde{C}_0$ heat exchanger (which by its nature cannot be optimal—see Sec. III) with the following design parameters:⁴

$$m=6.3 \text{ kg/s}, \quad m_0=6.923 \text{ kg/s}, \quad \tilde{\kappa}L=23\,515 \text{ W}, \\ \tilde{C}=4187 \text{ J/kg K}, \quad \tilde{C}_0=3810 \text{ J/kg K},$$

$$T(0)=283 \text{ K}, \quad T(L)=309.2 \text{ K},$$

$$T_0(0)=312.4 \text{ K}, \quad T_0(L)=338.6 \text{ K}.$$

Although the ratio T_0/T is not constant, it varies by only $\pm 0.4\%$. This heat exchanger produces entropy at the average rate $dS_{in}/dt = 212 \text{ W/K}$ as opposed to $dS_{in}/dt = 211 \text{ W/K}$ for optimal conditions for which $m_0^{\text{opt}} = 6.298 \text{ kg/s}$.

The perspective here is evaluating commonly used heating and cooling techniques in terms of their entropy production. In single-pass heat exchanger design the merits of the counterflow design have been recognized.¹ It has been shown here that one can also add the advantage of being able to achieve the heating/cooling strategy that minimizes entropy production.

¹W. M. Kays, *Compact Heat Exchangers*, 2nd ed. (McGraw-Hill, New York, 1964).

²P. Salamon, A. Nitzan, B. Andresen, and R. S. Berry, *Phys. Rev. A* **21**, 2115 (1980).

³B. Andresen and J. M. Gordon (unpublished).

⁴F. Kreith and M. S. Bohn, *Principles of Heat Transfer*, 4th ed. (Harper & Row, New York, 1986), Chap. 8, Sec. 8.4.