Determination of the Geoid Using Collocation

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1
1 Introduction

This assignment considers the computation of a geoid using the remove-restore principle along with collocation techniques and Stoke’s integration using Fast Fourier Transformation. The resulting geoid is represented by height anomalies $\zeta$, which means that the actual geoid computed is the quasi-geoid, which has a striking similarity to the traditional geoid represented by geoidal undulations $N$

$$N = h - H \quad \zeta = h - H^*$$

The geoid and quasi-geoid is related by simple formulas. The remove-restore principle also relies on a modern view on the determination of the geoid, from observations directly at the surface, which means that the gravity anomalies $\Delta g$ presented, is actually the free-air anomalies.

In the remove part long and medium wavelengths are removed using the EGM08 spherical harmonic model, while terrain corrections are handled by the RTM method using prism integration.

The quality of the final geoid can be evaluated by comparison to independent GPS-leveling data, which is given along with the assignment.

2 Fundamental Theory

In this some fundamental theory is introduced in order to fully understand the procedures of the computation. At first the classical view on the determination of the geoid is presented, then the modern view introduced by Molodensky and finally an interpretation of the classical gravity reductions in the modern theory.

2.1 The Classical Method

The geoidal undulation $N$ can be computed using Stoke’s integral

$$N = \frac{R}{4\pi \gamma_0} \int \int \Delta g S(\psi) d\sigma$$

Where $\Delta g$ is the gravity anomaly, $R$ is the radius of the earth, resulting from the spherical approximation, and $S$ is a function of the spherical distance $\psi$. This equation presupposes that the disturbing potential $T$ is harmonic outside the mass of the earth, which means that it satisfies the Poisson equation

$$\Delta T = \Delta (W - U) = 2\omega^2 - 2\omega^2 = 0$$

Here $W$ is the gravity potential and $U$ is the normal gravity potential. However, in order to use Stokes integral (2) and treat the problem as a boundary value problem, we need the anomalous potential $T$ to be harmonic on the boundary. The boundary being the geoid, this implies that there are no masses outside the geoid. Thus the classical problem is this: knowing the gravity anomalies $\Delta g$ on the surface of the earth, we need to make gravity reductions, not only shifting the points to the geoid, but also shifting or removing all residual masses inside the geoid.
However, these gravity reductions have an indirect effect on the geoid. By moving around external masses, we change the level surfaces and hence the geoid. This means that our point $P$ should not be reduced to a point $P_0$ on the geoid, but a point $P^c$ on a level surface we call the cogeoid. We can now compute the shape of the cogeoid by Stoke’s integral (2), using the reduced gravity anomalies

$$\triangle g^c = g^c - \gamma$$

From which we obtain the undulation $N^c$ of the cogeoid. Finally we have to consider the indirect effect and compute the geoidal undulation by

$$N = N^c + \delta N = N^c + \frac{\delta W}{\gamma}$$

Where $\delta W$ is the change in potential at the geoid. The problem however with gravity reductions is that we cannot know the density of the external masses everywhere, so we have to make assumptions about these densities.

### 2.2 The Modern View

The modern view on the determination of the geoid is based on ideas put forward by Molodensky. He invented a method of computing the geoid directly from observations on the surface of the earth. To do this the height anomaly was introduced

$$\zeta = h - H^*$$

where $h$ is the point $P$’s height above the ellipsoid and $H^*$ is the normal height of the surface point $P$. If we imagine us a continuous distribution of surface points, the combined surface of normal heights will make a surface referred to as the telluroid, i.e. a surface where the normal potential $U_Q$ is equal to the gravity potential $W_P$ of the surface points. Notice that the telluroid is not a level surface, because the surface of the earth is not a level surface.

The height anomaly $\zeta$ is this the difference between the telluroid and the surface, which corresponds to the geoidal undulation $N = h - H$, but uses the telluroid instead of the
The gravity anomaly $\Delta g$ is redefined into being the difference between the actual gravity measurement $g_P$ at the surface and the normal gravity $\gamma_Q$ on the telluroid

$$\Delta g = g_P - \gamma_Q$$

The normal gravity $\gamma_Q$ is computed using the normal gravity $\gamma_{Q_o}$ on the ellipsoid and then shifting this value upwards using free-air reduction. Thus this modern definition of the gravity anomaly is referred to as the free-air anomaly (throughout this paper I am going to use the term gravity anomaly, but I will be referring to the free-air anomaly). The surface given by a continuous distribution of height anomalies $\zeta$ above the ellipsoid, is referred to as the quasigeoid. It is not a level surface and has no physical meaning, but is of practical significance. What I am actually going to compute in this paper is the quasigeoid, but I will just refer to it as the geoid.

The normal height $H^*_P$ of a surface point is its elevation above the quasigeoid, just as the orthometric height $H$ is its height above the geoid.

2.3 Gravity Reductions in the Modern Theory

In this modern point of view introduced by Molodensky all measurements are referred to ground level, thus applying the traditional gravity reductions, such as the Bouguer or the isostatic reduction, has a different interpretation in this modern point of view.

By removing all residual masses above some reference level, inside the reference level, we change the potential and the gravity at the surface as

$$W^e = W - \delta W$$
$$g^e = g - \delta g$$

Here $W^e$ and $g^e$ denote the changed potential and gravity.

![Figure 3: The Effects of Gravity Reduction](image)

Because the potential on the surface change, so will the telluroid. When the telluroid changes the height anomalies will change, see figure 2. The new gravity anomaly on the surface point will be

$$\Delta g^e = \Delta g - \delta g - \frac{1}{\gamma} \frac{\partial \gamma}{\partial h} \delta W$$

---

1For a surface point $P$, the corresponding point $Q$ on the telluroid is found by Helmert projection, i.e. along the ellipsoidal normal
Thus, the reduced gravity anomaly \( \Delta g^c \) consists of the original surface anomaly \( \Delta g \) and two reductions:

- A direct effect \( \delta g \) resulting from moving the residual masses below the reference surface
- An indirect effect \( \frac{1}{7} \frac{\partial^2 g}{\partial R^2} \delta W \), which is caused by the reductions influence on the telluroid

The purpose of doing these reductions before computation is that the gravity anomalies \( \Delta g^c \) are smoother and more representative than the free-air anomalies \( \Delta g \) and thus can be interpolated more accurately.

### 3 The Remove-Restore Principle

Because the gravity anomalies \( \Delta g \) are correlated with height, the direct least-squares interpolation of gravity anomalies will lead to poor results in mountainous areas. Thus we need to remove these trends before applying the operation of least-squares interpolation. By least-squares interpolation we obtain a set of discrete gravity anomaly values, from which we can compute the geoid using Stoke’s integration (2). After computation the removed part must once again be restored; this is the remove-restore principle.

The remove-restore process aims at removing all major trends:

- Local trends producing Bouguer anomalies (local irregularities of elevation)
- Regional features producing topographic-isostatic anomalies (regional topographic effects)
- Global irregularities producing residual anomalies (these are expressed by an earth model)

In this assignment we will remove the global trends using the EGM08 and the local trends by using a Residual Terrain Model (RTM), where the effects of topographic irregularities are considered with respect to a mean elevation surface. This means that the potential can be computed by least-squares collocation, through the following steps:

- We need to remove the effects of all major trends on the gravity anomaly \( \Delta g \), this is done by removing the effects of an Earth Gravity Model \( \Delta g_{EGM} \) and the effects of a Residual Terrain Model \( \Delta g_{RTM} \)
- Using collocation we construct a grid of the residual gravity anomalies \( \Delta g_{res} = \Delta g - (\Delta g_{EGM} + \Delta g_{RTM}) \) and then, using Stoke’s integration, we compute the co-geoid \( \zeta^c \) from this residual gravity anomaly grid.
- The final geoid \( \zeta \) is obtained by restoring the indirect effect \( \zeta_{indirect} = \zeta_{EGM} + \zeta_{RTM} \) of the applied reductions

Mathematically this can be written as

\[
\zeta = L(\Delta g_{res}) + \zeta_{indirect}
\]  

(10)

Here \( L(\bullet) \) denotes the combined operation of linear least-squares collocation and Stoke’s integration.
3.1 Effects of the Earth Gravity Model

The Earth Gravitational Model (EGM08) is a gravitational model that consists of spherical harmonics of degree 2190, in this assignment however I only use the spherical harmonics of degree 340. The model is basically based on the harmonic properties of the disturbing potential \( T \), as described in subsection 2.1, which means that the potential can be represented by a spherical harmonic expansion

\[
T_{\text{EGM08}} = \frac{GM}{r} \sum_{n=2}^{N} \left( \frac{R}{r} \right)^n \sum_{m=0}^{n} \left( C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right) P_{nm} (\sin \phi)
\]  

(11)

but the EGM08 also gives us gravity anomalies \( \Delta g_{\text{EGM}} \) and height anomalies \( \zeta_{\text{EGM}} \), which is the quantities we want to use in the remove-restore operation.

3.2 Effects of the Residual Terrain Model

The Residual Terrain Modelling (RTM) is a terrain reduction, done relative to a mean elevation surface. The mean elevation surface is an average smooth surface of the topography where high frequencies have been filtered away.

Figure 4: Illustration of the mean elevation surface

In order to compute the effects from the residual topography with respect to this mean surface, we imagine the topography as made of homogeneous rectangular prisms. With respect to a station point \( P \), every prism’s effect on the observational value is considered. In this case we consider the effects on the gravity anomaly \( \Delta g \) by assuming a mean density of 2.67 g/cm\(^3\).

Because observations are more sensitive to effects in the area close to the station point, we use DTM grids with different density in grid points, to evaluate the effects on the observation point; this idea is illustrated by figure 5. The illustration shows that we use a detailed grid close to the observation point out to a radius \( R_1 \), then a less detailed grid out to a radius \( R_2 \).

Figure 5: Illustration of the concept of prism integration
Since the method allows for ground level points, to be below the reference surface, i.e. inside the masses of the topography, we need to apply a harmonic correction. This is because the gravity field modelling requires that observations can be derived from a harmonic function (outside the attracting masses, see Poisson’s equation (3)). Thus, to points below the mean elevation surface, a correction

\[ \Delta g_{hc} = -4\pi G \rho \delta h \]  

(12)
is applied. The distance \( \delta h \) is the distance between the sub-surface point and the mean elevation surface.

### 4 Presenting the Data

The data given to us along with the assignment is all in the area between latitude 31.5° N to 35° N and longitude 108° W to 105° W, which represents an area in New Mexico, USA. The area is illustrated by the red grid in figure 6

![Figure 6: Illustration of the area with data in New Mexico](image)

The data includes:

A **Digital Terrain Model (DTM)** containing normal heights \( H^* \) of the topography in grid form and spacing 0.00833333° × 0.00833333°

A **list of gravity stations** containing coordinates \((\phi, \lambda)\), normal heights \( H^* \) and gravity anomalies \( \Delta g \) in the form

<table>
<thead>
<tr>
<th>station nr</th>
<th>lat</th>
<th>lon</th>
<th>( H^* )</th>
<th>( \Delta g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
A list of 20 GPS stations containing coordinates $(\phi, \lambda)$, normal heights $H_{GPS}$, gravity anomalies $\Delta g_{GPS}$ and height anomalies $\zeta_{GPS}$ in the form:

<table>
<thead>
<tr>
<th>station nr</th>
<th>lat</th>
<th>lon</th>
<th>$H_{GPS}^*$</th>
<th>$\Delta g_{GPS}$</th>
<th>$\zeta_{GPS}$</th>
</tr>
</thead>
</table>

The data files are named ‘nmdtm’, ‘nmfa’ and ‘nmgpslev.ha’ respectively. The DTM is plotted in figure 7 along with the height of the gravity stations.

![Digital Terrain Model (nmdtm)](image1)  ![Gravity Stations (nmfa)](image2)

Figure 7: (left): DTM normal heights $H^*$, units in m, (right): Gravity station heights $H^*$, units in m

5 The Remove Part

In this section I will remove the major trends represented by $\Delta g_{EGM}$ from the earth model and by $\Delta g_{RTM}$ from local topography and regional features.

5.1 Extracting Data from EGM2008

From the Earth Model I chose to extract two data grids in the area under consideration, one grid containing the height anomalies $\zeta_{EGM}$ and one grid containing the gravity anomalies $\Delta g_{EGM}$. This is done using the GEOCOL17 program and is done with respect to the GRS80 reference ellipsoid. Since these values are to be subtracted from the corresponding ground level observations, we chose to extract each grid in two heights, meaning that we compute the four grids:

- EGM2008_TideFree_1000g.gri (gravity anomalies at 1000 m)
- EGM2008_TideFree_4000g.gri (gravity anomalies at 4000 m)
- EGM2008_TideFree_1000n.gri (height anomalies at 1000 m)
- EGM2008_TideFree_4000n.gri (height anomalies at 4000 m)
The fancy thing about doing so, is that we can create a sandwich grid form, from which we can determine the corresponding values anywhere in between the two grids, using linear interpolation. The values are thus harmonically continued in the space between the two grids, even though points will be inside the topographic masses.

By using a grid of $0.008333\degree \times 0.008333\degree$ resolution I achieved the statistical results shown in table 1.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle g_{EGM}$ at 1000 m</td>
<td>12.48 mgal</td>
<td>21.80 mgal</td>
<td>-21.91 mgal</td>
<td>80.47 mgal</td>
</tr>
<tr>
<td>$\triangle g_{EGM}$ at 4000 m</td>
<td>12.12 mgal</td>
<td>19.93 mgal</td>
<td>-18.57 mgal</td>
<td>73.42 mgal</td>
</tr>
<tr>
<td>$\zeta_{EGM}$ at 1000 m</td>
<td>-22.99 m</td>
<td>1.55 m</td>
<td>-26.27 m</td>
<td>-20.13 m</td>
</tr>
<tr>
<td>$\zeta_{EGM}$ at 4000 m</td>
<td>-23.03 m</td>
<td>1.51 m</td>
<td>-26.25 m</td>
<td>-20.28 m</td>
</tr>
</tbody>
</table>

In figure 8 I have plotted the gravity anomalies $\triangle g_{EGM}$ and the height anomalies $\zeta_{EGM}$ extracted from the EGM2008 at 1000 meters.

![Figure 8: (left): gravity anomalies $\triangle g_{EGM}$ from the EGM08 at 1000 meters, units of mGal (right): height anomalies $\zeta_{EGM}$ from the EGM08 at 1000 meters, units in m](image)

5.2 Removing EGM trends from the Gravity Data

We now want to remove the long wavelength trend of the EGM08 from our observations at ground level, which is gravity anomalies $\triangle g$ contained in the ‘NMFA’ file. These gravity anomalies $\triangle g$ corresponds to ground level observations at irregular distributed gravity stations. Thus, in order to subtract the EGM trend from the observations, we need to interpolate from our sandwich grid ‘EGM2008_g.gri’ to the coordinates of the stations. By coordinates I mean latitude, longitude and height; the height is the reason we made the sandwich grid.
This procedure is done using the GEOIP program, which in the same procedure can subtract the interpolated values from the observed gravity anomalies $\Delta g$ in the point list file 'NMFA'. The resulting file is named 'gravi.rdegm' and has the form

<table>
<thead>
<tr>
<th>station nr</th>
<th>lat</th>
<th>lon</th>
<th>$H^*$</th>
<th>$\Delta g - \Delta g_{EGM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

which means that we at each station ground point have the coordinates (lat, lon, $H^*$) and the gravity anomaly $\Delta g - \Delta g_{EGM}$ from where we have removed the EGM trend. Statistics can be seen in table 2 in subsection 5.4.

5.3 Handling the Digital Terrain Model

Along with this assignment I was handed a height data set in file 'nmdtm', which I from heron will refer to as the Digital Terrain Model (DTM). The DTM contains the normal heights $H^*$ of the topography in every grid point.

From the DTM we want to compute the effects of the external topography, this is referred to as Residual Terrain Modelling (RTM). Since RTM uses the method of prism integration explained in subsection 3.2, we need to thin our original DTM in order to make a less detailed grid. This is done using the program SELECT, which computes height values in the new grid points, by simple averaging of the data points closest to the grid point. Thus we make a grid in the usual area 31.5° to 35° N, 108° to 105° W with grid spacing 0.05° × 0.05° and name the file 'DEM5.gri'.

The mean elevation surface is computed from the 'DEM5.gri'-grid using the TCGRID program. The mean elevation surface is averaged in every $2 \times 2$ cell of the 'DEM5.gri'-grid and the low-pass filtering is obtained in a moving-average window of $7 \times 7$ cells. The mean elevation surface grid is named 'DEM_REF7.gri' and is plotted next to the thinned DTM grid 'DEM5.gri' in figure 9.

Figure 9: (left): Thinned DTM with grid spacing 0.05° × 0.05°, units in m (right): Mean elevation surface computed from the thinned DTM grid and low-pass filtered, units in m
5.4 Effects of the Residual Terrain Model

In order to compute the RTM effects $\triangle g_{RTM}$ we use the program TC. We use the DTM 'NMDTM.gri' as the detailed elevation grid, the thinning grid 'DEM.gri' as the coarse elevation grid and the mean elevation surface grid 'DEM5_REF7.gri' as the reference elevation grid. As the inner and outer radius $I$ use $r_1 = 12$ km and $r_2 = 999$ km. We want to compute the RTM effects at the gravity stations, so we use the 'gravi.rdegm' file as the station list file. The 'gravi.rdegm' file was the EGM effects $\triangle g_{EGM}$ subtracted from the original observational values $\triangle g$. The program TC allows us to add the RTM effects to the values $\triangle g - \triangle g_{EGM}$ already in the station list file, so that the resulting file 'gravi.rd' has the form

\[
\text{station nr lat lon } H^* \quad \triangle g - (\triangle g_{EGM} + \triangle g_{RTM})
\]

This means that our original gravitational observations $\triangle g$ has now been subtracted the major trends mentioned in section 3 and that the co-geoid can be computed using least-squares collocation.

Table 2: Statistical results of the gravity anomalies

<table>
<thead>
<tr>
<th>Gravity anomaly</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle g$</td>
<td>9.18 mgal</td>
<td>30.41 mgal</td>
<td>-58.70 mgal</td>
<td>162.5 mgal</td>
</tr>
<tr>
<td>$\triangle g - \triangle g_{EGM}$</td>
<td>-3.63 mgal</td>
<td>21.32 mgal</td>
<td>-69.35 mgal</td>
<td>116.7 mgal</td>
</tr>
<tr>
<td>$\triangle g - (\triangle g_{EGM} + \triangle g_{RTM})$</td>
<td>-0.50 mgal</td>
<td>11.67 mgal</td>
<td>-32.27 mgal</td>
<td>42.62 mgal</td>
</tr>
</tbody>
</table>

The statistics of the resulting 'gravi.rd' file can be seen in table 2. Notice how the distribution of gravity anomalies have been made more smooth with smaller standard deviation and less deviating minimum and maximum values. This makes the residual gravity anomalies $\triangle g_{res} = \triangle g - (\triangle g_{EGM} + \triangle g_{RTM})$ better for collocation in the next section.

6 Computation of the Co-Geoid

We are now set to compute the co-geoid. First we use the program GEOGRID to create a grid of gravity anomalies in the entire area. The reduced gravity anomalies $\triangle g_{res}$, contained at station points in the 'gravi.rd' file, is interpolated to all grid points using least-squares collocation. The resulting grid file is called 'gravi_rd.gri' and is plotted in figure 10.

The co-geoid can now be computed from Stoke’s integral. Using the program SPFOUR, this can be done using Fast Fourier Transformation (FFT), which is an algorithm for computing the Fourier transformation at discrete points. The resulting grid file is named 'zeta_rd.gri' and is plotted in figure 10.

7 The Restore Part: Consideration of the Indirect Effect

After computation of the co-geoid, we want to consider the effects of computing from reduced gravity values $\triangle g_{res}$. Considering the effect of EGM we return to the data extracted from the EGM in subsection 5.1.

We created a sandwich grid of the height anomalies $\zeta$ from grids at 1000 m and 4000 m. We now want to use this sandwich grid model to create a grid of EGM height anomalies.
\( \zeta_{\text{EGM}} \) at grid points corresponding to the thinned DTM file 'DEM5.gri'. This is done using the program GEOIP, which uses interpolation to compute the height anomalies \( \zeta_{\text{EGM}} \) at grid points given in the 'DEM5.gri' file. We name the output file 'EGM08n_topo.gri' and the statistics of this grid file can be seen in table 3.

<table>
<thead>
<tr>
<th>File name</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_{\text{EGM}} )</td>
<td>-23.01 m</td>
<td>1.54 m</td>
<td>-26.26 m</td>
<td>-20.18 m</td>
</tr>
<tr>
<td>( \zeta_{\text{RTM}} )</td>
<td>0.00 m</td>
<td>0.21 m</td>
<td>-0.42 m</td>
<td>1.04 m</td>
</tr>
</tbody>
</table>

We also need to consider the indirect effect of the RTM. This is done using the program TCFOUR, which computes the RTM effect \( \zeta_{\text{RTM}} \) of the residual topography on the height anomalies. This is done using the thinned DTM file 'DEM5.gri' and the mean elevation surface grid 'DEM_REF7.gri'. The output file is named 'zeta_rtm.gri' and its statistics can be seen in table 3.

Now, in order to restore the indirect effect and compute the geoid, we need to add the EGM effects \( \zeta_{\text{EGM}} \) ('EGM08n_topo.gri') and the RTM effects \( \zeta_{\text{RTM}} \) ('zeta_rtm.gri') to the computed co-geoid \( \zeta_{\text{res}} \) ('zeta_rd.gri'). The geoid is thus given by

\[
\zeta = \zeta_{\text{res}} + (\zeta_{\text{EGM}} + \zeta_{\text{RTM}})
\]

Starting with the RTM effects, all we need to do is add the two grids 'zeta_rd.gri' and 'zeta_rtm.gri', which can be done using the program GCOMB. The output file is named 'pip_zeta.gri' and is plotted in figure 11.

Next, we need to add the EGM effects, which is done by interpolation, since the grid spacing are not the same. Interpolation can be done using the program GEOIP and the output file 'geoid.gri' is our final geoid. It is plotted in figure 11. By comparing the final
geoid, figure 11 right, with the co-geoid, figure 10 right, we observe the effects of the remove part. Residual height anomalies $\zeta_{res}$ ranging from -0.5 to 0.5 m, while the final height anomalies $\zeta$ range from -26 to 20 m.

8 Independent Quality Control Using GPS stations

From the data file 'NMGPSLEV.HA' containing GPS stations with normal heights $H^*$ and height anomalies $\zeta_{GPS}$ we can make an independent quality control of our final geoid contained in file 'geoid.gri'.

This is done by interpolating the height anomalies $\zeta$ from the geoid grid, to the station points and subtracting the interpolated values from those in the GPS station list file. This is done using the program GEOIP and the statistics is seen in table 4.

<table>
<thead>
<tr>
<th>Table 4: Results of the quality control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>0.179 m</td>
</tr>
</tbody>
</table>

This procedure is independent because the height anomalies $\zeta_{GPS}$ in the GPS station list file, is obtained independently from GPS levelling.

8.1 The Effect of the Truncation Degree in the EGM

Since I have already made all the input files, it is easy for me to repeat the full procedure of computing the geoid. Changing the truncation degree of the EGM08 harmonic expansion, I have repeated the entire procedure for different truncation degrees.

The results of the independent quality control using GPS stations can be seen in table 5. The best result is obtained using a truncation degree of 340, which results in a standard deviation of only 43 mm.
Table 5: Quality control using various truncation degree

<table>
<thead>
<tr>
<th>Truncation degree</th>
<th>Mean</th>
<th>Std.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>320</td>
<td>0.181 m</td>
<td>0.049 m</td>
</tr>
<tr>
<td>330</td>
<td>0.181 m</td>
<td>0.049 m</td>
</tr>
<tr>
<td>340</td>
<td>0.179 m</td>
<td>0.043 m</td>
</tr>
<tr>
<td>350</td>
<td>0.177 m</td>
<td>0.046 m</td>
</tr>
<tr>
<td>360</td>
<td>0.180 m</td>
<td>0.048 m</td>
</tr>
<tr>
<td>2190</td>
<td>0.187 m</td>
<td>0.058 m</td>
</tr>
</tbody>
</table>

8.2 Kernel Modification of Stoke’s Integral

The Stoke’s integral

\[ N = \frac{R}{4\pi\gamma_0} \int \Delta g S(\psi) \, d\sigma \]  

involves the spherical Stokes kernel

\[ S(\psi) = \sum_{n=2}^{\infty} \frac{2n + 1}{n - 1} P_n(\cos \psi) \]  

which can be replaced by the Wong-Gore modified spheroidal Stokes kernel

\[ S_{\text{mod}}(\psi) = S(\psi) - \sum_{n=2}^{N_2} \alpha(n) \frac{2n + 1}{n - 1} P_n(\cos \psi) \]  

Traditionally the Stoke’s integral is evaluated from global data coverage. The incomplete global coverage in our computations has an impact on the spherical approximations in Stoke’s formula. Thus, the purpose of such kernel modifications is to reduce this impact from the low-frequency domain of the formula. The kernel modification is thus a kind of high-pass filtering performed to suppress the low-frequency errors until a degree \(N_1\). This is done by the coefficient \(\alpha(n)\) in (16) with

\[ \alpha(n) = \begin{cases} 
1 & \text{for } 2 \leq n \leq N_1 \\
\frac{N_2 - n}{N_2 - N_1} & \text{for } N_1 < n \leq N_2 \\
0 & \text{for } N_2 < n \leq N 
\end{cases} \]  

This ensures that the domain \(2 \leq n \leq N_1\) is suppressed, then a gradual linear transition in the domain \(N_1 < n \leq N_2\) and no modification in the domain \(N_2 < n \leq N\).

This kernel modification is used in computing the cogeoid from the residual gravity anomalies \(\Delta g_{\text{res}}\) in section 6 using the FFT technique. By varying the transition domain from \(N_1\) to \(N_2\) we affect the computation of the geoid.

Table 6: Quality control using various kernel modification parameters

<table>
<thead>
<tr>
<th>Degree</th>
<th>Mean</th>
<th>Std.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.154 m</td>
<td>0.057 m</td>
</tr>
<tr>
<td>40-50</td>
<td>0.159 m</td>
<td>0.054 m</td>
</tr>
<tr>
<td>60-70</td>
<td>0.167 m</td>
<td>0.049 m</td>
</tr>
<tr>
<td>80-90</td>
<td>0.179 m</td>
<td>0.043 m</td>
</tr>
<tr>
<td>100-110</td>
<td>0.196 m</td>
<td>0.040 m</td>
</tr>
<tr>
<td>120-130</td>
<td>0.214 m</td>
<td>0.043 m</td>
</tr>
</tbody>
</table>
In table 6 the result of varying the transition domain can be seen. The mean and standard deviation of the table is the final geoid fit to the GPS stations, which again is used for a quality check. The table shows the lowest standard deviation using a transition domain with $N_1 = 100$ and $N_2 = 110$. This choice of band reduces the standard deviation to 40 mm.

### 8.3 Systematic Errors: Correlation with Height

To check for systematic errors I have plotted the difference from the GPS quality control as a function of station height, this plot is seen in figure 12. From the figure we see that no significant correlation pattern can be observed.

![Figure 12](image)

Figure 12: Distribution of difference using GPS quality control with respect to station height

### 9 Conclusions

We arrived at a final geoid with a standard deviation of 43 mm, which I can only compare to the Auvergne case where a deviation of 29 mm was achieved, according to the paper: *Geoid determination in the mountains using ultra-high resolution spherical harmonic models - the Auvergne case*, handed out during class. Compared to this result, the one obtained here is not as good, but not massively bad either. In the final section we investigated the influence of truncation degree in the harmonic expansion of the EGM08, which showed a best fit at a truncation degree of 340. We also studied the effect of Kernel modification, which turned out to give best results, using a transition band from degree 100 to 110. Finally we checked for correlation with height, which revealed no significant correlation.
A  Input Files for Gravsoft Programs

A.1  EGM

```
# Gravity grid at 1000 meter
geoco17 <<!
ffftffftf
S - GRS80
EGM
3.98004415014 6178136.3 0.0 340 f f f f f f
EGM2008_to2300_TideFree
ffft
31.5 35.0 -108.0 -105.0 0.00833333 0.00833333 13 -1 1000.0 t t t
EGM2008_TideFree_1000g.gri
!

# Gravity grid at 4000 meter
geoco17 <<!
ffftffftf
S - GRS80
EGM
3.98004415014 6178136.3 0.0 340 f f f f f f
EGM2008_to2300_TideFree
t f f f
31.5 35.0 -108.0 -105.0 0.00833333 0.00833333 18 -1 4000.0 t t t
EGM2008_TideFree_4000g.gri
!

# The sandwich double grid model for gravity

copy EGM2008_TideFree_1000g.gri EGM2008_TideFree_4000g.gri EGM2008_TideFree_g.gri

# Height anomaly at 1000 meter
geoco17 <<!
ffftffftf
S - GRS80
EGM
3.98004415014 6178136.3 0.0 340 f f f f f f
EGM2008_to2300_TideFree
t f f f
31.5 35.0 -108.0 -105.0 0.00833333 0.00833333 11 -1 1000.0 t t t
EGM2008_TideFree_1000n.gri
!

# Height anomaly at 4000 meter
geoco17 <<!
ffftffftf
S - GRS80
EGM
3.98004415014 6178136.3 0.0 340 f f f f f f
EGM2008_to2300_TideFree
t f f f
31.5 35.0 -108.0 -105.0 0.00833333 0.00833333 11 -1 4000.0 t t t
EGM2008_TideFree_4000n.gri
!

# The sandwich double grid model for height anomaly

copy EGM2008_TideFree_1000n.gri EGM2008_TideFree_4000n.gri EGM2008_TideFree_n.gri

# Making tables with statistics

copy EGM2008_TideFree_1000g.gri in1.gri

copy EGM2008_TideFree_4000g.gri in2.gri

gcomb "in1.gri" "in2.gri"

```
A.2 DEM

DEM.job

```bash
# Thinning the grid
select <<!
NDTV.grd
DEM5.grd
3 7 1
31.5 35.0 -108.0 -105.0 0.05 0.05
!

# Creating reference grid
	cgrid <<!
DEM5.grd
DEM_REF7.grd
0 0 0 0
2 2 7 7
!
```

A.3 Remove

remove.job

```bash
# Interpolating from sandwich grid to points from point file
geosip <<!
...\DEM\GEO2008_TideFree_g.g
grav1_redeg
311 0 0 0
MPM.dat
1
1000 4000
!

# Adding the RM effects
tc <<!
grav1_redeg
..\DEM\MUTN.grd
..\DEM\DEM5.grd
..\DEM\DEM_REF7.grd
grav1_rd
5 4 1 3 2.67
31.5 35 -108 -105
12 999
!
```

A.4 Computing Co-Geoid

cogeoid.job

```bash
# Creating grid by interpolation
geogrid <<!
..\Remove\grav1.rd
grav1_rd.grd
1 1 5 0 1
15.0 0.5
13.0 0.5
!

# Computing the co-geoid by FFT using Stoke's formula
spfour <<!
grav1_rd.grd
Beta_0_g.gri
1 1 3 0 999
80 90
31.5 -108 100 100 0
!```
A.5  Restore

```
# Computing height anomalies in grid points
geolp <!
..: EGM08L_TideFree.m.g
EGM08L_topo.gr1
117 0 0 f
..: DEM08D_topo.gr1
1000 4000 1 f

# Effect of topography on geoid
tcfour <!
..: DEM08D_topo.gr1
..: DEM08D_topo.gr1
zeta_rtm.gr1
5 5 200 5
11.5 -10 100 140 1 f

# Making tables with statistics
copy EGM08L_topo.gr1 in1.gr1
copy zeta_rtm.gr1 in2.gr1
gcomb <!
in1.gr1
in2.gr1
diff -q
1 1

del in1.gr1
del in2.gr1
```

A.6  Computing Geoid

```
# Restoring the RM effects
gcomb <!
zeta_rdm.gr1
..: in1.gr1
..: zeta_rtm.gr1
pfl_zeta.gr1
5 5

# Restoring the EGM effects
geolp <!
..: EGM08L_topo.gr1
ggeo1d.gr1
16 0 0 f
pfl_zeta.gr1
```

A.7  GPS leveling

```
# Height anomaly differences geoid and gps
geolp <!
..: compute_geoid.gr1
ggeo1d.gr1_diff
11.0 0 f
pfl_GPSLEV.WA
1 1
```