

Geometric Explanation for Newtonian Gravity

!!!Draft!!!

A Revision of Newtonian Mechanics

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October 29. 2006

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1. Abstract

When studying Newtonian Mechanics we see, how mechanical forces are explained as causes of acceleration in inertial systems. In his 2nd law, Newton gave a formula for the impressed force. At the same time, Newton explained gravity as a mechanical force and gave a formula for gravity. In situations where the gravity pull is constant, that is when radius to the gravity center is constant, Newton's 2nd law is used on gravity, and the letter 'g' is often used as acceleration. The question arises: Is gravity a mechanical force? I'll show, that the Newtonian explanation is a misunderstanding of gravity, and that gravity is not like other mechanical forces.

2. Mechanical Force

2.1 Definition

A mechanical force is a force, that can be measured with a NewtonMeter.

A *NewtonMeter* is an apparatus often found in physics labs. Basically it's a spring, and often areas of red and white along a stick indicate the amount of force.

2.2 Postulate

The mechanics of a mechanical force is the same in all systems.

By this I mean, that when a mechanical force acts on a mass, the consequence should be the same, no matter where the mass is located. If in one situation at one location a mechanical force is the cause of an acceleration, then the result should be the same and of the same magnitude in any situation and location.

3. Three Situations

To study the cause and action of mechanical forces, I look at three situations, where a mechanical force results in acceleration of a mass:

- Constant Force in an Inertial System. I'll explore, how a constant force acts on a mass in an inertial system.
- Circular Motion in an Inertial System. I'll explore, how the force is working on a mass to force it into a circular motion within an inertial system.
- The Force of Gravity. I'll explore, how the force of gravity is the cause of motion near a heavy object.

4. Newtonian Mechanics

I'll start with, what is explained in Newtonian Mechanics.

4.1 An Inertial System

Newton's 1st law (the law of inertia) says:

A body remains in its state of rest or in uniform linear motion as long as no external forces act to change that state.

So we'll think of a situation, where a little mass is located far away from all other masses and no external force act on that mass. An inertial system could be an otherwise empty universe with only one little mass in it.

4.2 Constant Force in an Inertial System

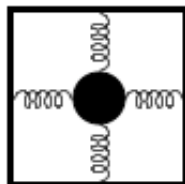
If we now act on the little mass with an external mechanical force, Newton's 2nd law say, that the force will be the cause of an acceleration according to the formula

$$\vec{F} = m\vec{a}$$

Now, who should judge, if the mass is accelerated in an empty universe? To solve this problem, let's have two observers, whose masses are so small, that they are negligible. If one observer is following the little mass and the other is left behind, they can agree on any acceleration coming from the force acting on the mass.

But then the next problem is raised. How can we judge, if a force is acting on an observer or not? If one observer is attached to the little mass, he has to be able to measure somehow, if a force is acting on the mass or not. If a force is acting on the mass, each observer will see the other accelerate away from him, but who is being accelerated, and who is at rest (or in uniform linear motion)?

To be able to tell, if an impressed force is at play, let's introduce a physical experiment. Imagine a transparent box with a metal sphere inside. Like a little ball. Let's give the sphere a mass of 1 kg, and let's attach the sphere to each of the six sides of the box with six springs, one to each side. If no mechanical force acts on the box, the sphere will be at rest at the center of the box. If an impressed force act, the sphere will move away from the center of the box for the period, while the force act. This shouldn't be difficult to imagine and also try out. The box looks like this:

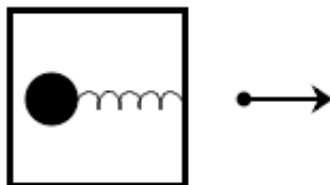


(This box works like a 3D NewtonMeter. It can measure an impressed mechanical force in any direction.)

This experiment will be a good method to figure out, if an external force acts on the mass. Actually this box with a sphere of 1 kg inside could be the mass, the observer is following. To simplify the experiment, only one spring is needed to attach the sphere to only one side of the box. Then the observer just have to rotate the box in a way, so the spring point in the direction of the impressed force, then the sphere will hang from the box with the help of the spring. In center position, it'll look like this:



And when an impressed force acts on a box, it'll look like this:



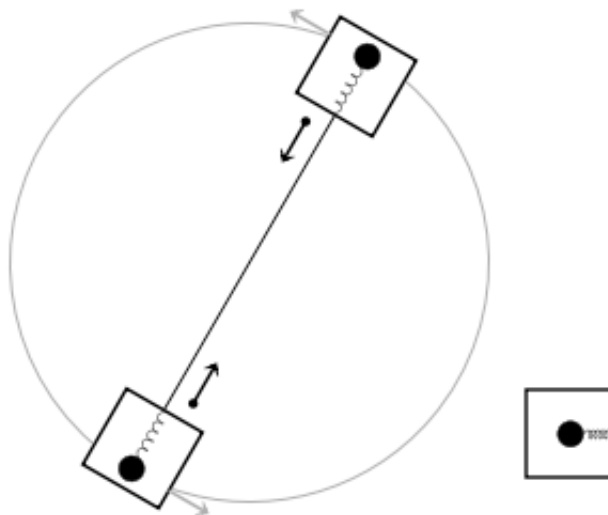
The arrow points in the direction of the impressed force and therefore also of the acceleration.

So now we give each observer a box, and the observer will be able to tell, if an external force is acting on the box by looking at the position of the sphere inside. If no external force act, the sphere will be at rest at the center of the box. If an impressed external force act, the sphere will move away from the center, and by rotating the box, the observer will be able to tell, in what direction the force act.

Let's imagine a mechanical force act on one box. The other observer will look at his box and say that no force is acting on him. When he looks at the other observer, he'll see, that he is moving away from him with constant acceleration. The box being accelerated will show, that an external force act on it, because the sphere inside the box is at a position away from the center, and the spring points in the direction of the force and acceleration.

4.3 Circular Motion in an Inertial System

Now let's look at a situation, where boxes are in circular motion. I use two boxes connected with a string, so the system is in balance.



The grey arrows illustrate the velocity, and the black arrows pointing in toward the center of motion are the forces. This situation also finely illustrates Newton's 3rd law:

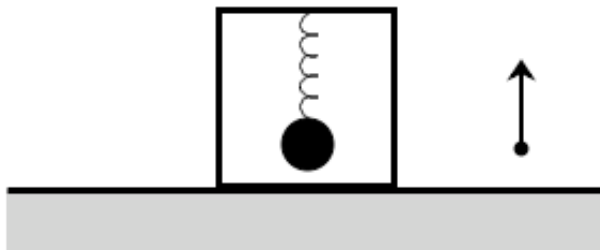
For every action, there is an equal and opposite reaction.

The box at rest next to the ones moving will show that no external force is acting on it. Its sphere will be at rest at the center of the box at rest. On the other hand will a box in circular motion show, that an external force act on it and the direction of the force is in towards the center of

circular motion. Both an observer following a box in motion, and another observer at rest with his box will agree on this.

4.4 The Force of Gravity

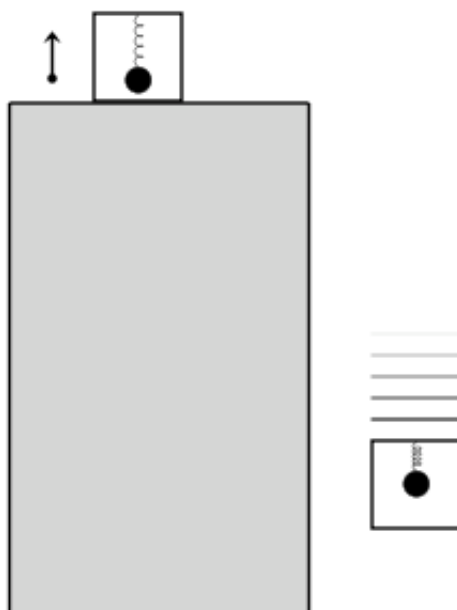
Now let's look at a box with a sphere standing on solid ground on the planet Earth. What will we see?



The sphere inside is not at the center position, it's down near the bottom of the box. Then, because of our initial postulate, we have to agree, that an impressed external force is acting on the box, and that it is pointing up! This force is the one called the *normal force*, and it's the only force acting on the box.

But the box is standing still!?

Like with the other two examples, we have to judge any motion of the box in relation to an observer, where no impressed force is at play. How can we have a box with a sphere positioned at the center of the box, if we're near the planet Earth? Let's assume that the two observers are on the top of a tower. One observer will keep his position there, and the other observer jumps out from the tower.



The one jumping out will now start to accelerate towards the center of the planet. (Let's keep air resistance out of the experiment.) While he's falling, he'll see the sphere in his box being located at the center of the box. [1] If he looks up, he'll see the other observer move away from him with increasing speed. And they'll both agree, that the box at the top of the tower will have its sphere located at the bottom of the box, and that the spring points in the direction of the impressed force acting on the box at the tower, and in the direction of the acceleration. That direction is up!

An observer in free fall near the box at the top of the tower will measure one acceleration of the box at the tower. The same observer at a later time will measure a higher acceleration (as long he's above the surface of the Earth), because he's further down, where the gravitational pull is stronger. If an observer in free fall was positioned above the tower, he'll measure a lower acceleration of the box at the top of the tower. But no observer in free fall will measure any external mechanical force on him, because the sphere in his own box is positioned at the center of the box. All observers will agree on the magnitude of the force needed to withstand the

gravitational pull at any given radius away from the center of the gravitational pull. They'll all see the sphere in the box at the top of the tower having a position near the bottom of the box.

5. What's wrong?

It seems from the last situation, that a mass not being influenced by an impressed force will accelerate, and the other two situations say the precise opposite. Something is wrong!

The problem is that we introduced a heavy mass (the planet Earth) to the scene. Let's look at the first situation with constant force far away from any other mass. The fact is, that however far away from a planet in an otherwise empty universe you are, the gravity pull from the planet is still there. Maybe it's tiny, but it's there. Even if you only had two peas at rest in opposite ends of an empty universe, they'll accelerate against each other and one day, they'll meet. If the two peas are at an initial position 15 billion light years from each other, and no external forces act on them, they'll meet one day. It might take long, but they'll meet.

So there are no inertial systems!

The physics, we study, should tell us about the universe, we live in. Our Universe! There are masses in The Universe, and you're influenced by those masses, wherever you are.

6. New Mechanics

Newton's 1st law has no physical meaning in our Universe. I'll therefore introduce an alternative:

6.1 The Law of Gravitational Acceleration

A body will accelerate towards the center of gravitational pull, if no external force acts to change that.

That's about the opposite of Newton's 1st law. Regarding Newton's 2nd law, it's valid for mechanical forces, but **not** for gravitational pull, because gravitational pull is not a mechanical force!

6.2 Gravity is not a Mechanical Force

Let's look at Newton's 2nd law again:

$$\vec{F} = m\vec{a}$$

That equal sign doesn't mean that force and acceleration are two sides of the same thing. What it means is, that a force can be the cause of acceleration. But this is not the case with gravitational pull, because when you're accelerated in towards the center of a big mass by the gravitational pull in a free fall, you can't measure any force. (This is only true, if the falling object is of point-size, otherwise you'll get tidal forces.)

The left side of the equal sign talks about dynamic, the right side about kinematic. Maybe the problem with Newtonian Gravity is that gravity is looked at as dynamic like any mechanical force. But it's of a different dynamic. Gravity is about the environment, where the kinematics take place.

We can say that the Newtonian explanation of gravity is the same kind of misunderstanding as that of the centrifugal force in a circular motion. Like there is no centrifugal force pointing out in a circular motion, there is no gravitational force pointing down near a big mass producing a gravitational pull. The pull is not a force, it's in the environment, where our masses are in free fall or standing still, if we act on them with a force pointing away from the gravitational center.

6.3 The Geometry of Space

It's not my intension to introduce the concept of space-time and curvation of that, because that's explained by Einstein and his general relativity. What I'm after is a better explanation of gravity as introduced by Newton. So I'll let the term **absolute time** as Newton explained it still be valid, and only look for a geometric explanation for the curvation of space.

The gravitational pull is explained in this Newton equation:

$$F = -G \frac{mM}{r^2}, \quad r = \sqrt{x^2 + y^2 + z^2}$$

Actually it's wrong to specify it as a force (using the letter F) as proven above. For now, let's specify the product of mass and gravitational acceleration:

$$mg = -G \frac{mM}{r^2}, \quad r = \sqrt{x^2 + y^2 + z^2}$$

So we have three spatial coordinates x , y and z , and together they can be used to tell the radius from the center of a gravitational pull.

The equation talks about two masses m and M , and if I eliminate m :

$$mg = -G \frac{mM}{r^2}$$

$$\Leftrightarrow g = -G \frac{M}{r^2}$$

I'll get the amount of acceleration g at a distance r from a mass M . G is the gravitational constant, and for a certain mass, M is also constant. So the gravitational pull is on the form:

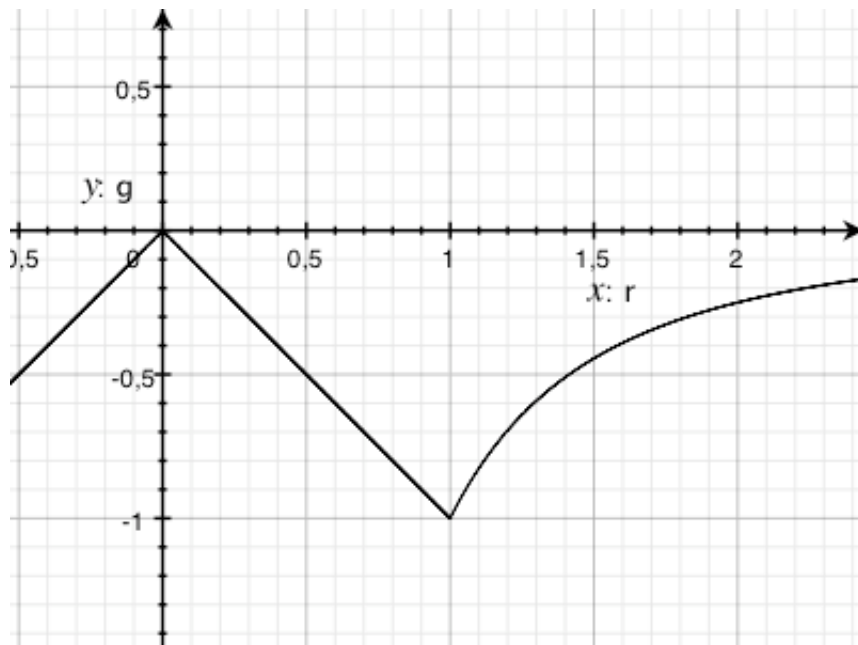
$$g = \frac{C}{r^2}$$

, where C is a negative constant.

The gravitational pull of a sphere like the Earth is largest at the surface of the sphere. Moving into the Earth and the pull will be smaller until the center of the Earth, where it'll be zero. For homogeneous spheres:

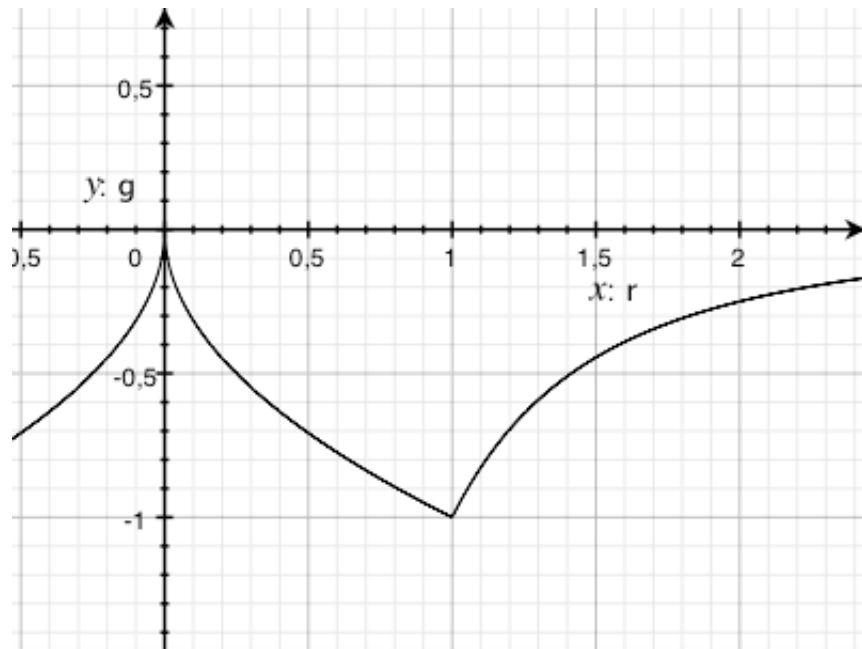
$$g = \begin{cases} -\frac{GM}{r^2} & r \geq \rho \\ -r & r < \rho \end{cases}$$

, where ρ is the radius of the sphere.

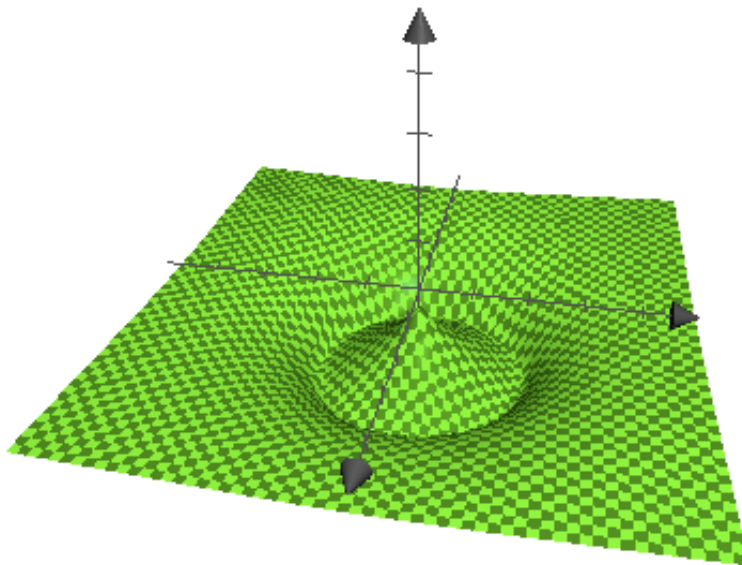


In reality, the density will raise, the closer to the center, you get. So for real spheres like the Earth, the Sun and other large spheres in our Universe, it'll be more like this:

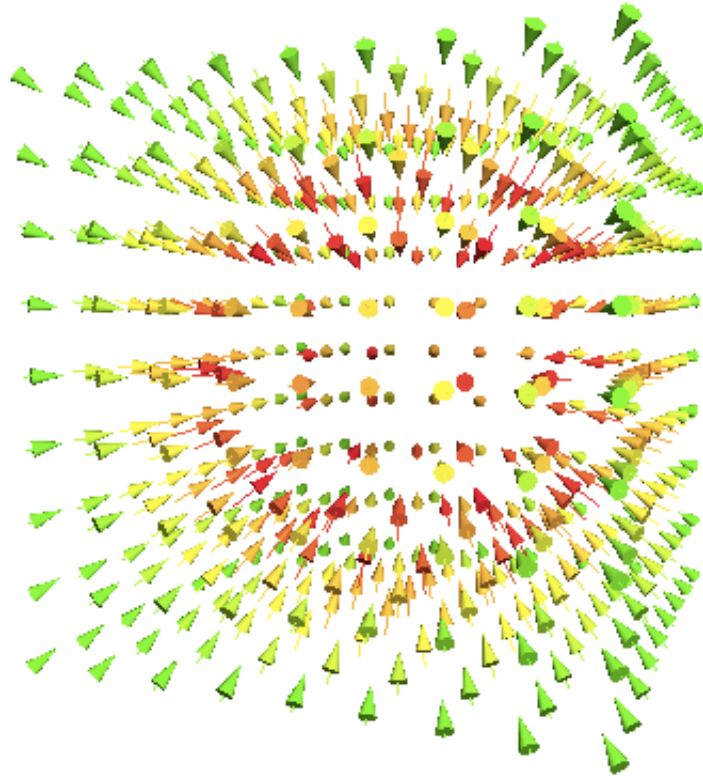
$$g = \begin{cases} -\frac{GM}{r^2} & r \geq \rho \\ -\sqrt{r} & r < \rho \end{cases}$$



The pull as a function of radius can be visualized as a 2D surface curved in 3D like this:



Sometimes we talk about a gravity field. [2] Such a field can be visualized as arrows in a 3D space:



In this image, the larger and redder the arrows are, the stronger the gravitational pull is. In the middle, the arrows form a sphere, and that illustrates the sphere producing the gravity field.

6.4 A Function of One Variable

To explore the curvature of space, we think of gravity as a function of one variable, the radius r :

$$g(r) = -\frac{GM}{r^2}$$

Let's now think of a curve, where the x-component of the normal to any point on the curve shall give us g . The normal is a vector perpendicular to the tangent for any point on the curve. The x-component can be calculated as $g_1 \cos(\theta(r)) \sin(\theta(r))$ where g_1 is an imaginary pull down perpendicular to the x-axis. So I have:

$$-g_1 \cos(\theta(r)) \sin(\theta(r)) = -\frac{GM}{r^2}$$

, where $\theta(r)$ is the angle between the tangent for any point on the curve and the x-axis. This is the same as the angle between the normal for any point and the y-axis. θ depends on r , as the curvature of space depends on the distance from the center of the gravitational pull. I find θ :

$$\begin{aligned} -g_1 \cos(\theta(r)) \sin(\theta(r)) &= -\frac{GM}{r^2} \\ \Leftrightarrow \frac{1}{2} g_1 \sin(2\theta(r)) &= \frac{GM}{r^2} \\ \Leftrightarrow \theta(r) &= \frac{1}{2} \sin^{-1} \left(\frac{2GM}{g_1 r^2} \right) \end{aligned}$$

If I name the function for the curve, I'm after, Υ , I have this relation between θ and the derivative of Υ :

$$\begin{aligned} \tan(\theta(r)) &= \Upsilon'(r) \\ \Rightarrow \Upsilon'(r) &= \tan \left(\frac{1}{2} \sin^{-1} \left(\frac{2GM}{g_1 r^2} \right) \right) \end{aligned}$$

$$\Rightarrow \Gamma'(r) = \tan\left(\frac{1}{2} \sin^{-1}\left(\frac{g_1 r^2}{2GM}\right)\right)$$

If I measure r in units of ρ , that is the radius of the sphere producing the gravitational pull, I get for

$$r = 1\rho:$$

$$\begin{aligned}\theta(\rho) &= \frac{1}{2} \sin^{-1}\left(\frac{2GM}{g_1 \rho^2}\right) \\ \Leftrightarrow g_1 \sin(2\theta(\rho)) &= 2\left(\frac{GM}{\rho^2}\right)\end{aligned}$$

If I set $\theta(\rho) = \pi/4$ and see, that the right side is 2 times g at the surface of the sphere:

$$\begin{aligned}\theta(\rho) = \frac{\pi}{4} \quad \wedge \quad g &= \frac{GM}{\rho^2} \\ \Rightarrow g_1 \sin\left(2 \cdot \frac{\pi}{4}\right) &= 2g \\ \Leftrightarrow g_1 &= 2g \\ \Rightarrow \Upsilon'(r) &= \tan\left(\frac{1}{2} \sin^{-1}\left(\frac{GM}{gr^2}\right)\right) \\ \Leftrightarrow \Upsilon'(r) &= \tan\left(\frac{1}{2} \sin^{-1}\left(\frac{\rho^2}{r^2}\right)\right)\end{aligned}$$

What this mean is, that I now have the derivative of a curve producing a pull in along the x-axis according to the gravitational pull. The curve is described by a pull down along the y-axis of $2g$. g here is the gravitational pull at the surface of a sphere, and in the situation of the Earth:
 $g \approx 9,82m/s^2$.

The curve itself is described as:

$$\Upsilon(r) = \int \tan\left(\frac{1}{2} \sin^{-1}\left(\frac{\rho^2}{r^2}\right)\right) dr$$

There is no simple solution to this equation. To explore the function, it's possible to calculate values for it's derivative for different values of r :

$$\begin{aligned}\Upsilon'(\rho) &= 1 \\ \Upsilon'(2\rho) &\approx 0,127 \\ \lim_{r \rightarrow \infty} \Upsilon'(r) &= 0\end{aligned}$$

7. Footnotes

1. The observation, that the outcome of a physical experiment is independent of the velocity of the free-falling reference frame, in which it is performed, was discovered by A. Einstein in 1907, is known as the *equivalence principle* and led Einstein to formulate his theory of general relativity.
2. If gravity can be explained by field theories like the other natural forces (the electromagnetic force, the weak nuclear force and the strong nuclear force), there should be a particle transferring the force, and we can talk about a gravitational field in the concept of quantum-field theories. That particle is known as the graviton, and it hasn't been discovered at the date of this paper. If gravity can't be described with field-theories like the other natural forces, then maybe we shouldn't talk about a gravitational field.