

G

The Gravitational Constant
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1. The Value of G

$$G = (6.6742 \pm 0.0010) \times 10^{-11} \frac{m^3}{s^2 kg}$$

2. Two Alternatives

There are two main alternatives to the understanding of gravity.

2.1 Field Theories

One way to see gravity is like the other natural forces and describe it with a field theory. To that field belongs a particle known as the graviton. The value of G should then probably be explained by some particle physics.

The graviton hasn't been discovered as of this writing.

2.2 Pure Geometry

The other way to see gravity is as a pure geometric phenomena. This is what Einstein introduced with his theory of general relativity in 1915-1916.

If gravity is purely related to the geometry of the Universe, then the value of G would probably be closely related to the constitution of the Universe.

3. G Units

G is measured in

$$\frac{m^3}{s^2 kg}$$

If the value of G is related to values for the Universe, I need to put those values of the Universe together in an equation, so G is calculated with the right units.

4. Universal Values

4.1 Speed of Light

$$c \approx 2.99792458 \times 10^8 \frac{m}{s}$$

4.2 Age of the Universe

The data from WMAP [1] specify the age of the Universe to:

$$T_U \approx 13.7 \times 10^9 \text{years} \approx 4.3233912 \times 10^{17} \text{s}$$

4.3 Size of the Universe

This is a bit difficult, because there are several opinions. As the matter at the greatest visible distance from us move away from us close to the speed of light, we can say, the radius of the Universe is:

$$R_U \approx 13.7 \times 10^9 \text{lightyears} \approx 1.2961201 \times 10^{26} \text{m}$$

But that light coming from the matter at the greatest visible distance from us is old. The matter has moved further away, since the light was sent out.

There also is the theory of inflation. If that is true, the Universe might be a lot bigger.

4.4 Weight of the Universe

Again according to WMAP data, the mass density in the Universe is:

$$\Omega = 9.9 \times 10^{-30} \frac{\text{g}}{\text{cm}^3} = 9.9 \times 10^{-27} \frac{\text{kg}}{\text{m}^3}$$

This gives this mass for the Universe:

$$\begin{aligned} M_U &= \Omega \cdot 4/3\pi R_U^3 \\ \Leftrightarrow M_U &\approx 9.0294135 \times 10^{52} \text{kg} \end{aligned}$$

5. Equations

G can be calculated in several ways and give the right units:

$$G_U = \frac{R_U^2}{T_U M_U} \cdot c$$

$$G_U = \frac{R_U}{M_U} \cdot c^2$$

$$G_U = \frac{R_U^3}{T_U^2 M_U}$$

All these give:

$$G_U \approx 1.2901111 \times 10^{-10} \frac{\text{m}^3}{\text{s}^2 \text{kg}}$$

The right value for G is:

$$G = (6.6742 \pm 0.0010) \times 10^{-11} \frac{\text{m}^3}{\text{s}^2 \text{kg}}$$

The error is:

$$G_U/G \approx 1.932982$$

Not bad!

If I e.g. change the radius of the Universe from

$$R_U \approx 1.2961201 \times 10^{26} \text{m}$$

to

$$R_U \approx 1.8020183 \times 10^{26} \text{m}$$

(which also change the mass of the Universe), I get

$$G_U = 6.6742 \times 10^{-11} \frac{m^3}{s^2 kg}$$

The exact value measured. It's more likely, that all of the values for the Universe should be changed a bit, and not just the Radius. With only minor modifications of the universal values, it's possible to get the right value for G .

So maybe G really is a constant (or variable) closely related to the constitution of our Universe.

6. Alternative Equations

Maybe the volume of the Universe should be used?

$$G_U = \frac{4/3\pi R_U^3}{T_U^2 M_U}$$

$$G_U = \frac{4/3\pi R_U^3}{T_U^2 \Omega \cdot 4/3\pi R_U^3}$$

$$\Leftrightarrow G_U = \frac{1}{\Omega T_U^2}$$

Rewrite of

$$G_U = \frac{R_U}{M_U} \cdot c^2$$

$$\Leftrightarrow G_U = \frac{R_U c^2}{\Omega \frac{4}{3}\pi R_U^3}$$

$$\Leftrightarrow G_U = \frac{3c^2}{4\pi\Omega R_U^2}$$

Rewrite of

$$G_U = \frac{R_U^3}{T_U^2 M_U}$$

$$\Leftrightarrow G_U = \frac{R_U^3}{T_U^2 \cdot \Omega \cdot \frac{4}{3}\pi R_U^3}$$

$$\Leftrightarrow G_U = (4/3\pi\Omega T_U^2)^{-1}$$

7. Footnotes

1. WMAP at NASA: <http://map.gsfc.nasa.gov/>