Eur. J. Phys. 4 (1983) 235–239. Printed in Northern Ireland

Quantum correlations and measurements

A R Mackintosh and J Jensen

H C Ørsted Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

Received 14 September 1983, in final form 21 November 1983

Abstract It is demonstrated that the collapse of the wavefunction in the quantum measurement process is reflected in the two-particle correlation function for the electron spin, which can in principle be studied by a triple Stern–Gerlach experiment. The change in momentum of a Stern–Gerlach magnet is itself sufficient to allow the determination of the electron spin, subject to the uncertainty principle, if the coherence of the two components of the spin is destroyed by the magnetic field. Similar considerations apply to the two-particle correlation function for the photon polarisation, and two-photon atomic cascades can be used in practice to investigate the measurement process and determine the extension of a photon wavepacket. The irreversibility of quantum mechanical measurements and its relation to the uncertainty principle is briefly considered.

Resumé Det vises at bølgefunktionens kollaps ved den kvantemekaniske måleproces i princippet kan studeres i et tredobbelt Stern–Gerlach eksperiment gennem en undersøgelse af to-partikel-korrelationsfunktionen for elektronernes spin. Ødelægger det magnetiske felt kohærensen mellem en elektrons to spintilstande, vil Stern–Gerlach-magnetens impulsændring, under hensyntagen til usikkerhedspricippet, i sig selv tillade en bestemmelse af elektronens spin. Lignende forhold gør sig gældende for to-partikel-korrelationsfunktionen for fotoners polarisering, og et atomart to-foton kaskadehenfald kan udnyttes i praksis til et studie af måleprocessen og til en bestemmelse af fotonbølgepakkens udstrækning. Irreversibiliteten af den kvantemekaniske måleproces og dens relation til usikkerhedsprincippet diskuteres kort.

1. Introduction

More than half a century after the formulation of quantum mechanics, two fundamental problems remain the subject of debate and controversy. The incompleteness of the quantum description and the acausality implied by the uncertainty principle have led to attempts to construct more complete and causal theories based on hidden variables. Furthermore the process of measurement, which plays a very special role in the epistemology of the quantum theory, remains in some respects obscure. Both of these questions can in principle be elucidated by studying the two-particle correlations of electron spin or photon polarisation.

In an earlier article in this journal, devoted primarily to electron spin, one of us (Mackintosh 1983, referred to hereafter as I) discussed hidden-variable theories and Bell's inequalities. Since the pioneering experiments of Freedman and Clauser (1972) there has been great interest in using measurements of the correlations between the polarisations of the two photons emitted in atomic cascade processes to test such theories. The overwhelming majority of such experiments give results in agreement with the predictions of quantum mechanics, violating Bell's inequalities and hence contradicting local hidden-variable theories. Most recently Aspect *et al* (1982) have shown that Bell's inequalities are also violated when the directions of the measured polarisations are determined during the flight of the photons. The primary purpose of this paper is to show that a straightforward extension of these two-photon correlation experiments can cast light on the quantum mechanical measurement process and the associated collapse of the wavefunction, providing incidentally an alternative way of measuring directly the spatial extent of a photon wavepacket.

We begin in the next section by discussing two-electron spin correlations, showing that these are modified by the measurement process, and how this modification can in principle be studied by a triple Stern–Gerlach experiment. The conditions for a measurement to occur are considered, and it is shown that the change in momentum of the Stern–Gerlach magnet is in itself sufficient to allow the determination of the electron spin, subject to the

0143-0807/83/040235+05$2.25

uncertainty principle, if the coherence of the two components of the spinor is destroyed by the magnetic field. Practical methods are then described for studying the measurement process through two-photon correlations, by introducing a further polariser into the standard experimental arrangement. The connection between the coherence of the wavefunction, the uncertainty principle and the measurement process in such an experiment is considered. In the final section we summarise the results of our analysis and discuss the irreversibility inherent in quantum mechanical measurements.

2. The Stern–Gerlach experiment

To illustrate the principles involved, we will first consider the triple Stern–Gerlach experiment depicted in figure 1, which is an extension of the hypothetical experiment of Bohm (1951), discussed in I. Two electrons, initially in a singlet state, are ejected from a source in opposite directions. The magnet designated M is used to measure the spin of electron 1, and hence collapse the two-electron spin function, and the effect of this measurement is observed through the correlations between the components of the spins of the two electrons in different directions, determined by means of the two magnets C_1 and C_2. A similar arrangement has recently been suggested by Hartmann (1983) in connection with a discussion of Bell's inequalities.

We suppose that the magnetic field of M is initially turned off and the spin function is therefore, in the conventional notation of I

$$|S\rangle_i = 2^{-1/2}(|1+\rangle|2-\rangle - |1-\rangle|2+\rangle). \quad (1)$$

The components of the spins of the two electrons are measured with C_1 and C_2, rotated through angles θ_1 and θ_2 respectively relative to the z direction and in the plane normal to the electron velocity. For each electron two possible results $\sigma = \pm 1$ (spin up or down) may be obtained. The expectation value of the product $\sigma_1\sigma_2$ may be obtained by elementary transformation theory, as in I, and is

$$E_{|S\rangle_i}(\sigma_1\sigma_2) = -\cos(\theta_1 - \theta_2) = -\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2. \quad (2)$$

The magnetic field of M is now increased and the spin of electron 1 in the z direction thereby measured, so that the two-electron spin function collapses to

$$|S\rangle_f = |1+\rangle|2-\rangle \quad \text{or} \quad |1-\rangle|2+\rangle. \quad (3)$$

The expectation value of $\sigma_1\sigma_2$ is the same in both these cases, and may be similarly calculated to be

$$E_{|S\rangle_f}(\sigma_1\sigma_2) = -\cos\theta_1\cos\theta_2. \quad (4)$$

This is the mean of the correlation function (2) for a total spin $S = 0$ and that for $S = 1$; $S_z = 0$, illustrating that the total angular momentum of the electron system is changed by the measurement. The effect of the measurement on the spin correlation function is to eliminate the sine term, specifying a reference axis by the direction of the magnetic field. We note that, ignoring possible spatial separations of the different spin states, the measurement process cannot be registered by using either C_1 or C_2 separately, since each will record an equal number of up- and down-spin electrons in all orientations, irrespective of whether the wavefunction has collapsed or not. However, a study of the spin correlations can reveal whether or not a measurement has been made. The collapse of the spin wavefunction in the measurement process can also in principle be observed by passing a single particle through three successive rotatable Stern–Gerlach magnets. It may readily be shown that a measurement of the spin in the second magnet changes the correlations between measurements in the first and third magnets.

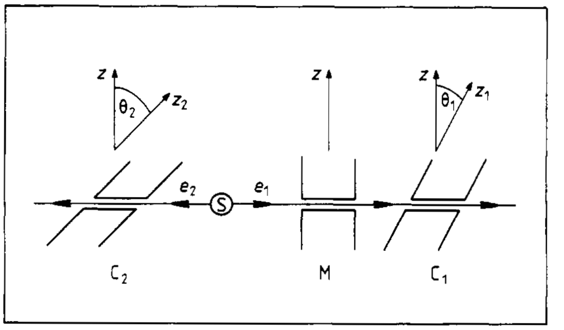

Figure 1 Schematic arrangement of the triple Stern–Gerlach experiment. Two electrons in a singlet state are emitted from the source S. The spin of one of them may be measured by means of the inhomogeneous field in the magnet M, and the consequences of this measurement investigated through correlations between the spins determined by the magnets C_1 and C_2, which may be rotated in the plane normal to the paths of the electrons.

It is therefore natural to enquire how large the magnetic field-gradient in M must be before a measurement can be said to have occurred. When passing through the inhomogeneous magnetic field, the electron is subjected to a force $\pm\mu_B\,\partial B/\partial z$, for $\sigma_z = \pm 1$ respectively, and if it spends a time T in the magnet it acquires a momentum $\pm\mu_B T\,\partial B/\partial z$ in the z direction. The magnet acquires an equal and opposite momentum and this can in principle be measured, and the electron spin thereby specified, provided that such a measurement does not conflict with the uncertainty principle. A measurement can therefore be made if the change in the magnet's momentum is greater than the uncertainty in this momentum or, by the uncertainty relation, if

$$\mu_B T \frac{\partial B}{\partial z} \frac{\Delta z}{h} > 1 \tag{5}$$

where Δz is the uncertainty in the position of the magnet in the z direction. This uncertainty in the magnet's position gives rise to an uncertainty ΔB in the field, which in turn leads to an uncertainty $\mu_B \Delta BT/\hbar$ in the phases of the components of the spinor, due to the passage of the electron through the field. The inequality (5) is thus also the condition that this phase uncertainty is greater than 2π, and hence that the coherence between the two components of the spinor is destroyed by the field.

The spin is not, of course, normally measured by observing the change of momentum of the magnet, but rather by recording the separated beams of spin-up and spin-down electrons on a screen, or by counters. The condition that the angular separation of the two beams is greater than the angular width of either due to diffraction by the magnet aperture may then be shown (Baym 1969) to be the same as equation (5), except that the uncertainty in the magnet position Δz is replaced by the width d of its aperture. The splitting of the beams is not in itself a sufficient condition for a measurement of the spin and, when equation (5) is not satisfied, as is normally the case, the beams will maintain their phase coherence, even though they are split. The magnet then imposes a linear variation in phase on the wavefront, of opposite sign for the two spins, but this phase difference may in principle be cancelled by a further magnet, restoring the original phase relation between the two components of the spinor.

The phase of the wavefunction could be investigated by a variant of the experiment discussed in I, in which a neutron beam is split into two and later recombined, while maintaining its coherence, in a Bonse–Hart spectrometer. If one of the beams were passed through an inhomogeneous magnetic field, the resulting interference fringes would be modified in a manner reflecting the relative phases of the spinor components.

3. Photon correlations

Although the possibility of performing the Stern–Gerlach experiment illustrated in figure 1 is remote, that of figure 2, which is an extension of the method of Aspect *et al* (1982), is in principle very similar and in practice quite feasible. An atomic cascade is used to produce two photons with correlated polarisations, a measurement of one of them is made by means of the polarising cube M and suitably disposed mirrors, and the consequent collapse of the wavefunction is registered by the polarising cubes C_1 and C_2, which can be rotated in the plane normal to the photon paths, and associated electronics. For example, the Ca $4p^2\,{}^1S_0 \rightarrow 4s4p\,{}^1P_1 \rightarrow 4s^2\,{}^1S_0$ cascade, in which a yellow and a blue photon are successively emitted, is particularly suitable for such experiments.

When the photons are emitted in opposite directions, the symmetry of the atomic wavefunctions ensures that their polarisations, although unspecified, must be the same. The two-photon wavefunction may therefore be written

$$|P\rangle_i = 2^{-1/2}(|1x\rangle|2x\rangle + |1y\rangle|2y\rangle) \tag{6}$$

and a measurement of the polarisation collapses this to

$$|P\rangle_f = |1x\rangle|2x\rangle \quad \text{or} \quad |1y\rangle|2y\rangle. \tag{7}$$

We define p_1, to be ± 1 if the measurement on photon 1 with the polarimeter C_1, oriented at an angle θ_1, registers x_1 and y_1 polarisation respectively, and similarly for photon 2. The photon wavefunction transforms like a vector and it may hence be readily shown that the expectation values of p_1p_2 are

$$\begin{aligned} E_{|p\rangle_i}(p_1p_2) &= \cos 2(\theta_1 - \theta_2) \\ &= \cos 2\theta_1 \cos 2\theta_2 + \sin 2\theta_1 \sin 2\theta_2 \end{aligned} \tag{8}$$

and

$$E_{|p\rangle_f}(p_1p_2) = \cos 2\theta_1 \cos 2\theta_2. \tag{9}$$

The measurement of the polarisation of the photon and the associated collapse of the wavefunction therefore again eliminates the sine term.

It is interesting to consider under which circumstances we can expect to measure the correlation functions (8) and (9) which, for convenience, we will denote interfering and noninterfering respectively. If the coherence of the x and y polarised waves is maintained through the various reflections and transmissions, which can readily be accomplished in practice, then they can interfere coherently when recombined at the transmitting mirror m_3 and we expect to observe the interfering correlation function. On the other hand, if either path is blocked then the polarisation is specified and the noninterfering correlation function will result. The polarisation can also in principle be measured without absorbing either of the photons, since the path which a particular photon has taken can be determined by detecting the transfer of momentum, for example to mirror m_1, when it is reflected. It is straightforward to show (Baym 1969) that the condition that this momentum transfer be greater than the uncertainty in the mirror momentum, and hence measurable, implies that the uncertainty in the position of the mirror is greater than a wavelength. The coherence between the two polarisations is therefore destroyed by such a measurement and the noninterfering correlation function must be observed.

Even if this coherence is maintained, however, it might be argued that the polarisation can be measured, by making use of coincidences between the two photons. Because of the extra pathlength Δl which a y polarised photon must travel in M, it is

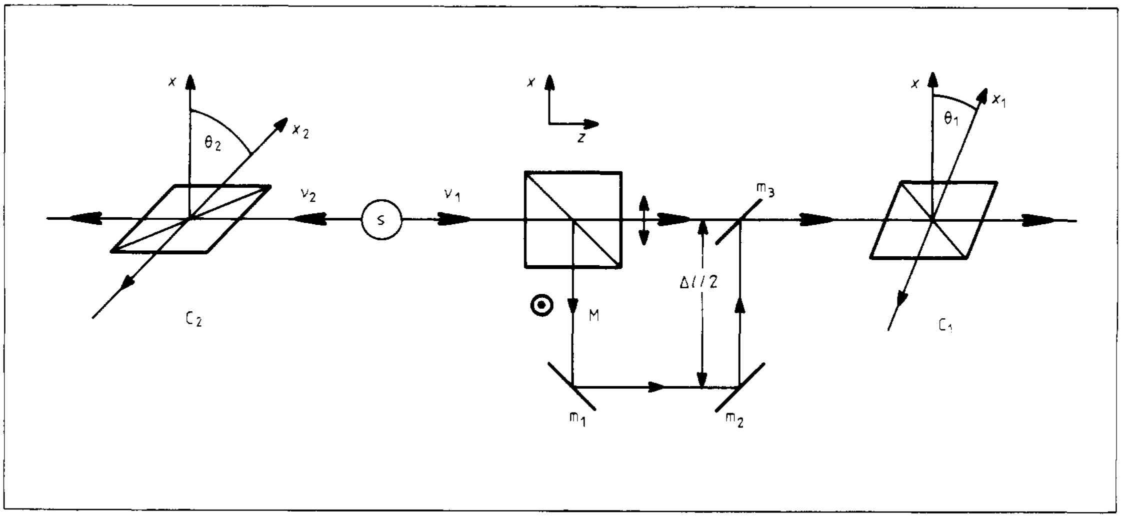

Figure 2 Schematic arrangement for measuring photon polarisations. Two photons with the same polarisation are emitted from the source S. The polarisation of one of them may be measured by the combination M of a cube polarimeter and mirrors, and the consequences of this measurement investigated through correlations between the polarisations determined by the cube polarimeters C_1 and C_2, which may be rotated in the plane normal to the paths of the photons.

delayed by a time $\Delta l/c$ and this delay can be registered in a coincidence measurement, thereby implicitly determining the polarisation. We are therefore faced with the apparent paradox that, even though the two waves can interfere coherently, the delay allows the measurement of the polarisation of photon 1 when it leaves M, and the noninterfering correlation function corresponding to the collapsed wavefunction must then be observed. This paradox may be resolved by recalling that the 4s4p $^1P_1$ intermediate level has a lifetime τ. If, for example, we assume that a blue photon passes through the polarimeter system M (which may be accomplished in practice with a filter), the polarisation can only be measured by the coincidence method if the delay $\Delta l/c > \tau$. However, the energy–time uncertainty principle implies that the photon wavepacket has an extension $c\tau$, and the condition for the measurement is just that this extension is less than Δl. If therefore a measurement can be made in this way, the wavepackets corresponding to the two polarisations do not overlap, because of their finite extensions and, since they cannot then interfere, the noninterfering correlation function results. It is therefore possible to change the form of the measured correlation function by mounting the mirrors m_1 and m_2 on slides and changing Δl. For small values, the interfering correlation function will be observed but, as Δl becomes comparable with $c\tau$, the sine term will gradually vanish, so that the noninterfering correlation function is observed when Δl is large. Since τ is approximately 4.5×10^{-9} s (Wiese *et al* 1969), $c\tau$ is about 1.4 m so that this experiment, which directly measures the extension of the photon wavepacket, should be feasible. We note that the extension of the yellow photon wavepackets is slightly less than that of the blue, because the finite lifetime of the $4p^2\ {}^1S_0$ initial state, which is about 10^{-9} s, also contributes to the uncertainty in their energy. If yellow photons are passed through the polarimeter system M, the decay of the sine term with Δl will therefore be somewhat faster, even though the condition for a polarisation measurement by the coincidence method, which only depends on the lifetime of the intermediate state, is unchanged. Hence the noninterfering correlation function may be observed even though no measurement of the polarisation has been made; such a measurement is a sufficient but not a necessary condition for observing the noninterfering correlation function (9). However the wavefunction is only irreversibly collapsed when the photons are registered in the detectors, since the overlap and interference of the two polarisation waves can, previous to this detection, be restored by a suitable arrangement of polarisers and mirrors.

The above discussion is closely related to Schrödinger's cat paradox. The passage of a photon through a specified arm of the polarisation measurement system M is supposed to trigger a gun which shoots an unfortunate cat. For an arbitrary polarisation, the wavefunction of the system of cat plus photon is a linear combination of states in which the cat is alive and dead, which Schrödinger regarded as paradoxical. The analysis of this experiment causes no difficulties when we realise that, in order to trigger the gun, a measurement must be made of the photon polarisation. If no measurement is made, the cat is safe. If a measurement is made in M, the cat may be lucky or unlucky, but is always unambiguously either alive or dead.

4. Measurements

We have thus seen that the quantum mechanical measurement process is very similar in the cases of electron spin and photon polarisation. The initial state of the quantum system is a coherent superposition of the eigenstates of the dynamical variable which is to be measured. The classical measuring apparatus destroys the coherence between these eigenstates, selecting one of them and determining which it is by a simultaneous registration of a macroscopic change in the apparatus. The conditions that the coherence is destroyed and that the change in the apparatus can be measured, subject to the uncertainty principle, are the same.

The concept of a macroscopic change in a classical measuring apparatus is frequently used in discussions of the measurement process, without any very precise definition. Our analysis indicates that a macroscopic change of, for example, the momentum in this context is one which is much greater than the uncertainty in the momentum of the apparatus, which is therefore classical in the sense that it is not limited by the uncertainty principle. The congruence of the conditions for collapsing the wavefunction and for making an observable change in the apparatus is presumably quite general for quantum measurements, and is another example of the internal consistency of the quantum theory.

The measurement process can be studied experimentally, and we have shown how this might be done in practice. The experimental possibilities for the spin are at present severely limited, but a searching investigation can be made of the measurement of the photon polarisation and the associated collapse of the wavefunction by studying the two-photon correlation functions. These experiments, which involve standard optical techniques with polarisers, mirrors and filters, appear to be quite feasible.

It is interesting to consider the connection between measurements and irreversibility in the quantum theory. The orthodox viewpoint, developed primarily by von Neumann (1932), and expounded for example by Wigner (1963), asserts that the change of a system with time may take two forms. The state vector evolves continuously with time according to Schrödinger's equation, as for example when an electron beam passes through a Stern–Gerlach magnet without the fulfillment of the inequality (5). This time-dependence is reversible in the same sense as the time-development of a classical system is reversible. The state vector may also change discontinuously, irreversibly and acausally, subject to the laws of probability, when a measurement is made on the system. Such an irreversible change occurs for example in a Stern–Gerlach magnet if equation (5) is satisfied. This irreversiblity is in contrast to classical mechanics and establishes a direction for time. The basic conundrum of measurement theory has been expressed by Bell (1975) as follows: 'So long as the wavepacket reduction is an essential component, and so long as we do not know exactly when and how it takes over from the Schrödinger equation, we do not have an exact and unambiguous formulation of our most fundamental physical theory'. We suggest that such a wavepacket reduction occurs whenever two systems interact sufficiently strongly that a measurable change is produced in the one which gives information on the quantum state of the other after the interaction. It is not necessary that the former system is a measuring apparatus in the conventional sense, nor that the change which it undergoes as a result of the interaction is actually observed. Our discussion indicates, not surprisingly, that the criterion for determining whether a change is measurable in this sense is supplied by the uncertainty relations. The role of the uncertainty principle as the fundamental element of the quantum theory extends therefore to determining whether or not an irreversible change has occurred when two systems interact, and hence to establishing the direction of time.

Acknowledgments
We have benefitted from illuminating discussions with Aage Bohr, P Voetmann Christiansen, J Henningsen, H Højgaard Jensen, B R Mottelson and E Veje.

References

Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804
Baym G 1969 *Lectures on Quantum Mechanics* (New York: Benjamin)
Bell J S 1975 *Helv. Phys. Acta* **48** 93
Bohm D 1951 *Quantum Theory* (Englewood Cliffs, NJ: Prentice-Hall)
Freedman S J and Clauser J F 1972 *Phys. Rev. Lett.* **28** 938
Hartmann K M 1983 *Phys. Lett.* **97A** 15
Mackintosh A R 1983 *Eur. J. Phys.* **4** 97
von Neumann J 1932 *Die mathematischen Grundlagen der Quantenmechanik* (Berlin: Springer)
Wiese W L, Smith M W and Miles B M 1969 *Atomic Transition Probabilities* NBS Reference Data
Wigner E P 1963 *Am. J. Phys.* **31** 6