AC Losses in Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10+x}$ Tapes and a 3.15-m-Long Single-Phase Cable

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Abstract—The alternating-current losses in superconducting multifilament BiSCCO-2223 tapes and a 3.15-m single-phase test cable were measured at 77 K using an electrical transport method. The cable had an inner diameter of 42 mm; it was composed of a single layer of 31 multifilament tapes and had a critical current of $I_c = 4.1$ kA. The measured losses of the tapes were found to be in good agreement with the Norris ellipse (NE) model. The losses of the cable were, for high currents, found to be bounded by the monoblock and independent NE models.

Index Terms—High-temperature superconductors (HTSs), loss measurement, magnetic losses, multifilamentary superconductors.

I. INTRODUCTION

SINCE the first high-temperature superconductor (HTS) was discovered in 1986, scientists have worked to improve the quality of industrial superconductors, making them technologically and economically competitive with copper conductors. HTS cables are strong candidates for ac-power transmission cables [1], and in several places, superconducting cables have been introduced on a trial basis [2]–[4].

The reduction of ac losses is critical for the commercialization of superconducting cables [5]. To reduce the ac losses, it is necessary to understand the processes governing them. So far, no satisfying general theoretical model for calculating ac losses in cables has been established, although several models for estimation of the loss for certain geometries have been published [6]–[9].

In this paper, the ac losses were measured on both superconducting single tapes and on a 3.15-m-long cable constructed from similar tapes. The superconducting tapes were Bismuth-

based multifilamentary HTS wire encased in a silver matrix and laminated with brass. The cable critical current was calculated to 4.1 kA based on measurements done on individual tapes.

II. MEASUREMENTS OF AC LOSSES IN A SINGLE TAPE

The critical current of the tapes used in the cable was measured by a standard four-probe direct-current method averaging both current directions and using the standard criterion defining I_c at a voltage drop of 1 μV/cm. Thirty one tapes from the same batch of tapes were used to construct the cable. From this batch of tapes, 13 samples were selected to be tested in detail in single tape configuration. The I_c of the tapes was found to be 132 ± 6 A.

The ac losses in the tapes were measured using the electrical method, being faster and more accurate than calorimetric ones [10], [11], using a 60-Hz ac. A lock-in amplifier extracted the first harmonic $I(t) = I_p \sin(\omega t)$ from the signal, giving a voltage drop over the tape of

$$U(t) = RI_p \sin(\omega t) + \left[LI_p\omega - \frac{I_p}{C\omega} \right] \cos(\omega t) \quad (1)$$

$$= U_R(t) + U_L(t). \quad (2)$$

Here, R is the electrical resistance, C the capacitance, and L is the inductance of the tape. In the second line, $U_R(t)$ is the sine term, and $U_L(t)$ is the cosine term.

The loss per unit time can now be calculated as

$$P = \frac{1}{T} \int_0^T I_p \sin(\omega t) \left(U_R(t) + U_L(t) \right) dt \quad (3)$$

$$= I_{\text{RMS}} U_{R,\text{RMS}}. \quad (4)$$

This holds for $T \gg (2\pi/\omega)$.

The experimental setup is shown in Fig. 1 and is similar to the one used in [12]. A Rogowski coil (i.e., RC 1) [13] with an integrator was used to measure the RMS value of the ac through the superconductor. The phase of the current was measured by another Rogowski coil (i.e., RC 2) connected to a lock-in amplifier that also received a reference signal from the ac source. The phase shift given by RC 2 was used to calibrate the reference signal such that the lock-in amplifier could compare this to the actual voltage characteristic and extract $U_{R,\text{RMS}}$ from the in-phase voltage drop. We carried out this measurement for different currents and determined the losses per unit length from (4) by dividing by the distance ℓ between the voltage contacts.
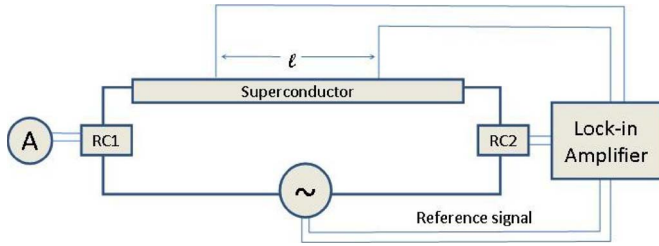
Fig. 1. Experimental setup for the measurements of ac loss in an HTS tape. The RMS current was measured with Rogowski coil RC 1, whereas its phase was measured using RC 2, as explained in the text. The resistive part of the voltage drop $U_{R,\mathrm{RMS}}$ was measured with the lock-in amplifier.
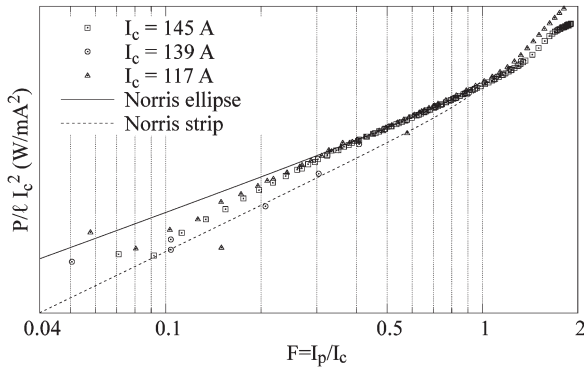

Fig. 2. AC losses of HTS tapes per second normalized by length L and I_c^2. The NE model is seen to fit the data best, particularly at high currents.

The procedure was repeated for three HTS tapes of different critical currents. The resistive part of the measured voltage drop was much smaller than the inductive part, particularly at low currents. The relative uncertainties of the measurements are therefore largest in the low-current domain, as is seen from the obvious deviations among measurements in Fig. 2.

III. MEASUREMENTS OF AC LOSSES IN THE SUPERCONDUCTING CABLE

A 3.15-m single-layer single-phase superconducting cable was constructed from 31 multifilament tapes taken from several different spools wound around a 42-mm diameter fiberglass former with a pitch of $\theta = 11.2°$ (see Fig. 3). Assuming that the current in the cable was evenly distributed among the tapes and neglecting the change in critical current from the magnetic field, we determined the cable $I_c = 4.1$ kA by multiplying the average critical current of an individual tape, i.e., 132 ± 6 A, by the number of tapes in the cable.

Using a method similar to the one for single tapes, the ac losses of the cable were measured for different currents at a frequency of 50 Hz. The results are displayed in Fig. 4.

Inhomogeneities in critical current densities of tapes, contact resistances, and local cable geometry will lead to systematic errors in the losses. These can be reduced with the use of voltage taps and wires regularly distributed around the cable, as described in [14] and [15].

Again, the resistive part of the voltage drop over the cable was much smaller than the inductive, particularly for small currents; thus, the uncertainties are most significant in the low-current domain.
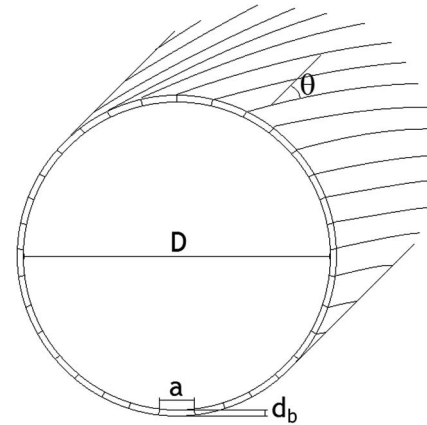

Fig. 3. Cross section of the superconducting cable used (for values of the parameters in the figure, see Table I).
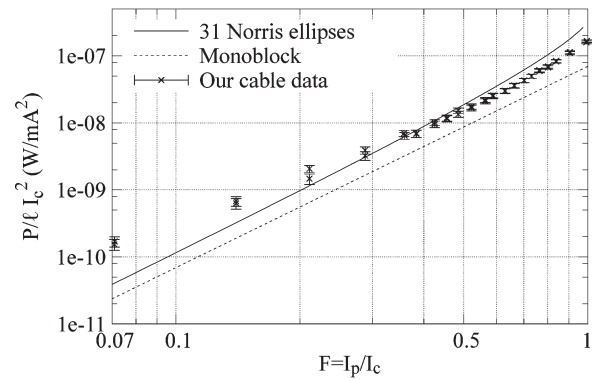

Fig. 4. AC losses of the HTS cable in units of loss per time normalized by length and I_c^2. The error bars are the uncertainties of the individual measurements. It is seen that the MB model has approximately the same dependence on F as the data but underestimates the values. The NEs give values closer to the data but changes much faster with F. Therefore, the models cannot accurately estimate the loss but only give upper and lower bounds for the loss at high currents.

The normalized ac loss in the cable at a relative peak current of $F = 0.995$ was measured to be 0.166 μW/A^2. This is around twice than that found in [12] on an eight-layer single-phase cable with a critical current of 3240 A but significantly lower than the losses measured on triaxial cables with critical currents of 6 and 4 kA in [16] and in [17], respectively. Note that the losses should not be compared directly due to significant differences in the cable geometries.

IV. MODELING AC LOSSES IN SINGLE HTS TAPES

In 1970, Norris presented models for calculating ac losses in superconductors with different geometries [6]. Today, these models are still widely used when ac losses are estimated using explicit expressions.

For a single HTS tape, the "Norris strip" (NS) and the "Norris ellipse" (NE) can be used to estimate the ac loss per length per cycle of the current

$$L_{NS} = \frac{I_c^2 \mu_0}{\pi} \left((1-F)\ln(1-F) + (1+F)\ln(1+F) - F^2 \right) \quad (5)$$

$$L_{NE} = I_c^2 \frac{\mu_0}{\pi} \left((1-F)\ln(1-F) + F\left(1 - \frac{F}{2}\right) \right) \quad (6)$$

where I_c is the critical current of the tape and $F = I_p/I_c$, where I_p is the peak current.

In both models, the tape is treated as if it were made up of solid superconducting material instead of individual filaments. Physically, this means that no magnetic field is allowed to circulate between the filaments in the silver matrix. This approximation agrees well with experiments [7].

Refined models have been developed based on other geometries, e.g., [8] and [9], but no analytical models have been as successful as the NS and NE models.

In Fig. 2, the measured losses for HTS tapes are plotted against the normalized NS and NE models. For peak currents larger than $0.3I_c$, the NS model predicts a loss much lower than experimentally measured. The NE model, on the other hand, agrees very well for peak currents larger than $0.3I_c$. For industrial purposes, high currents are the most interesting; furthermore, the uncertainty of the data is lowest for high currents. It is therefore concluded that the best estimation of the ac loss of a multifilament BiSCCO-2223 tape is obtained using the NE model.

V. MODELING AC-LOSS IN HTS CABLE

When the individual superconducting tapes are assembled to form a cable, the ac loss can be estimated in two different ways.

First, the N individual tapes can be considered to be independent. From (6), we derive that the total ac loss L_N will be

$$L_N = \frac{\mu_0 I_c^2}{N\pi\cos\theta}\left[(1-F)\ln(1-F) + F\left(1 - \frac{F}{2}\right)\right] \quad (7)$$

where I_c and F refer to the entire cable and $\cos\theta$ is the relation between the length of the tapes and the length of the cable.

Alternatively, the tightly packed tapes can be considered to be a seamless tube. Then, the cable can be regarded as a thin shell of superconducting material carrying a relative current F along the cable on the outer surface. This is the monoblock (MB) model giving the loss [18]

$$L_{MB} = \frac{\mu_0 I_c^2}{\pi h^2}\left[(1-Fh)\ln(1-Fh) + Fh\left(1 - \frac{Fh}{2}\right)\right] \quad (8)$$

where I_c and F again refer to the entire cable and h is defined from the diameter of the former D and the effective thickness of the tapes d as follows:

$$h = \frac{(D + 2d)^2 - D^2}{(D + 2d)^2} \approx 4\frac{d}{D} \ll 1. \quad (9)$$

The relevant values of D and d can be found in Table I.

Since h is small, we can use Taylor expand equation (8) around $Fh = 0$; this gives to the following leading order:

$$L_{MB} \approx \frac{\mu_0 I_c^2}{6\pi h^2}(Fh)^3. \quad (10)$$

In contrast with the case of individual tapes in (7), the loss in an MB depends explicitly on the effective thickness d of the superconducting tapes. This dependence indicates that the comparison with the MB model is going to depend sensitively on the quality of the superconducting tapes and, in particular, on the effective thickness of the superconducting layer.

TABLE I
PHYSICAL CHARACTERISTICS OF THE CABLE. HERE, D AND L ARE THE DIAMETER AND THE LENGTH OF THE FORMER, RESPECTIVELY; ℓ IS THE DISTANCE BETWEEN THE VOLTAGE CONTACTS; θ IS THE PITCH ANGLE; a AND d_b ARE THE WIDTH AND THICKNESS OF THE BRASS LAMINATE; d IS THE EFFECTIVE THICKNESS OF THE SUPERCONDUCTING LAYER; AND I_c IS THE EXPECTED CRITICAL CURRENT BASED ON THE AVERAGE CURRENT OF THE INDIVIDUAL TAPES

D [mm]	42 ± 2
L [cm]	315 ± 1
ℓ [cm]	231.9 ± 0.1
θ [°]	11.2 ± 0.7
a [mm]	4.1 ± 0.2
d_b [mm]	0.4 ± 0.05
d [mm]	0.22 ± 0.01
I_c [kA]	4.1 ± 0.1

Because the tapes are actually pitched, there will be a tangential component of the current, which according to the model in [19], will flow on the inner surface to expel the magnetic field from the superconducting material. Since this current is factor $\tan(\theta)$ less than F, the resulting loss will be on the order of $(\tan(\theta)Fh)^3$ and thus negligible for the angle θ in question here.

In Fig. 4, the measured losses of the cable are plotted against the normalized L_N and L_{MB} losses. The MB model has the right dependence on F but gives a loss approximately a factor of 2 lower than that measured. The model consisting of N independent tapes predicts for $F \approx 0.35$ a loss close to that measured, but for higher currents, it gives a loss approximately a factor of 1.5 higher than that measured.

This shows that the ac loss cannot be correctly calculated either using the MB model or a model of 31 independent tapes. However, these models give a lower and an upper bound to the actual loss, corresponding to the fact that the tapes in a real cable is neither independent nor completely adjoined.

Because measurements and models are all normalized by I_c^2, the comparison is independent of the value of the critical current of the cable. However, differences in critical currents of the individual tapes will result in an uneven magnetic field distribution around the cable. This will result in slightly higher measured losses than if all tapes had the same critical current.

VI. CONCLUSION

For currents higher than $F = 0.3$, the ac loss of superconducting $Bi_2Sr_2Ca_2Cu_3O_{10+x}$ tapes are best estimated from the NE model. A 3.15-m-long superconducting cable with a critical current of $I_c = 4.1$ kA was constructed from 31 tapes, and the ac loss was measured. For peak currents close to I_c, the measured value was two thirds of the loss arising from 31 independent tapes and twice the loss predicted by the MB model.

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