Condensed Matter Physics 2

Skriftlig eksamen - April 2011

2-timers skriftlig prøve. Sædvanlige hjælpemidler er tilladte (bøger, noter og lommeregner). Opgaverne må gerne besvares med blyant.

Problem 1: Ferromagnet. Atoms with spin $s = \frac{1}{2}$ are placed on an fcc lattice with lattice constant a (primitive lattice vectors with length $\frac{a}{\sqrt{2}}$) and volume V. The atomic spin at the ith site $\vec{s}_i$ interacts with its neighbours as described by the Heisenberg Hamiltonian:

$$\mathcal{H} = -\frac{1}{2}\sum_{i,j} J(i,j)\,\vec{s}_i \cdot \vec{s}_j \qquad (1)$$

$J(i,j) = J_1 > 0,$ if i and j are nearest neighbours
$J(i,j) = J_2 > 0,$ if i and j are next-nearest neighbours
$J(i,j) = 0,$ if i and j are not nearest or next-nearest neighbours

a) Find the ground state energy (the internal energy at zero temperature) of this spin system in terms of J_1, J_2, a, and V.

b) Show that the present spin system and the simple Ising model discussed by Marder in Section 24.4 become equivalent when applying the mean-field approximation. (Hint: assume the presence of an infinitesimal magnetic field along the z axis).

c) Utilize this equivalence for determining the mean-field value for the ordering temperature T_C of the present spin system.

Problem 2: Non-interacting spin-dimer system. The lattice describing the system has a basis that contains two identical atoms 1 and 2 with spins $\vec{s}_1$ and $\vec{s}_2$. The spins have $s = \frac{1}{2}$ and are coupled with each other. Any other spin interactions between different pairs or "dimers" are neglected. Hence, the spin Hamiltonian is

$$\mathcal{H} = -J\sum_{i=1}^{N} \vec{s}_1(i)\cdot\vec{s}_2(i) - g\mu_B\vec{H}\cdot\sum_{i=1}^{N}\Big[\vec{s}_1(i) + \vec{s}_2(i)\Big] \qquad (2)$$

in the presence of a magnetic field $\vec{H}$. The density of spins is twice the number of unit cells per unit volume, i.e. $n = 2N/V$.

a) Determine the eigenenergies for a single spin dimer in terms of J and H (choose the z axis to be parallel with $\vec{H}$).

b) What is the magnetic susceptibility $\chi = \frac{\partial M}{\partial H}\Big|_{H\to 0}$ of this system, if the interaction J is neglected?

c) Determine χ in the temperature range where $k_BT \ll |J|$ in the two cases $J > 0$ and $J < 0$ (use arguments rather than calculations in your answer).

(the problems are continuing on next page)

Problem 3: Anisotropic band electrons. The band energies of the conduction electrons in a crystal with orthorhombic symmetry are given by

$$\varepsilon(\vec{k}) = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 k_y^2}{2m_2} + \frac{\hbar^2 k_z^2}{2m_3} \tag{3}$$

where the x, y, and z axes are parallel with each one of the three orthorhombic lattice vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$. The density of the conduction electrons is n.

a) Describe the constant energy surface (Fermi surface) within reciprocal space.

b) Write down the electrical conductivity tensor $\overline{\overline{\sigma}}$ (at zero magnetic field) when assuming a constant relaxation time τ.
Use this result for determining the direction of the current density $\vec{j}$ when an electric field $\vec{E}$ is applied in a direction which is perpendicular to the z axis and makes an angle of 45° with the x axis.

c) The density of states for the anisotropic band electrons is the same as for free electrons, if the electron mass is replaced by an effective mass m^*. Determine m^* in terms of the three band masses m_1, m_2, and m_3.
The low-temperature specific heat is $c_V = \gamma T$. Derive the expression for the Sommerfeld constant γ in terms of the three band masses and the density n.

(opgavesættet er slut)