Sir William Rowan Hamilton

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born August 3/4, 1805, Dublin, Ireland
died September 2, 1865, Dublin

Sir William Rowan Hamilton, 1862. *The Royal Irish Academy, Dublin*

Irish mathematician who contributed to the development of optics, dynamics, and algebra—in particular, discovering the algebra of quaternions. His work proved significant for the development of quantum mechanics.

Hamilton was the son of a solicitor. He was educated by his uncle, James Hamilton, an Anglican priest with whom he lived from before the age of three until he entered college. An aptitude for languages was soon apparent: at five he was already making progress with Latin, Greek, and Hebrew, broadening his studies to include Arabic, Sanskrit, Persian, Syriac, French, and Italian before he was 12.

Hamilton was proficient in arithmetic at an early age. But a serious interest in mathematics was awakened on reading the *Analytic Geometry* of Bartholomew Lloyd at the age of 16. (Before that, his acquaintance with mathematics was limited to Euclid, sections of Isaac Newton's *Principia*, and introductory textbooks on algebra and optics.) Further reading included works of the French mathematicians Pierre-Simon Laplace and Joseph-Louis Lagrange.

Hamilton entered Trinity College, Dublin, in 1823. He excelled as an undergraduate not only in mathematics and physics but also in classics, while he continued with his own mathematical investigations. A substantial paper of his on optics was accepted for publication by the Royal Irish Academy in 1827. In the same year, while still an undergraduate, Hamilton was appointed professor of astronomy at Trinity College and Royal Astronomer of Ireland. His home thereafter was at Dunsink Observatory, a few miles outside Dublin.

Hamilton was deeply interested in literature and metaphysics, and he wrote poetry throughout his life. While touring England in 1827, he visited William Wordsworth. A friendship was immediately established, and they corresponded often thereafter. Hamilton also admired the poetry and metaphysical writings of Samuel Taylor Coleridge, whom he visited in 1832. Hamilton and Coleridge were both heavily influenced by the philosophical writings of Immanuel Kant.

Hamilton's first published mathematical paper, "Theory of Systems of Rays," begins by proving that a system of light rays filling a region of space can be focused down to a single point by a suitably curved mirror if and only if those light rays are orthogonal to some series of surfaces. Moreover, the latter property is preserved under reflection in any number of mirrors. Hamilton's innovation was to associate with such a system of rays a characteristic function, constant on each of the surfaces to which the rays are orthogonal,

which he employed in the mathematical investigation of the foci and caustics of reflected light.

The theory of the characteristic function of an optical system was further developed in three supplements. In the third of these, the characteristic function depends on the Cartesian coordinates of two points (initial and final) and measures the time taken for light to travel through the optical system from one to the other. If the form of this function is known, then basic properties of the optical system (such as the directions of the emergent rays) can easily be obtained. In applying his methods in 1832 to the study of the propagation of light in anisotropic media, in which the speed of light is dependent on the direction and polarization of the ray, Hamilton was led to a remarkable prediction: if a single ray of light is incident at certain angles on a face of a biaxial crystal (such as aragonite), then the refracted light will form a hollow cone.

Hamilton's colleague Humphrey Lloyd, professor of natural philosophy at Trinity College, sought to verify this prediction experimentally. Lloyd had difficulty obtaining a crystal of aragonite of sufficient size and purity, but eventually he was able to observe this phenomenon of conical refraction. This discovery excited considerable interest within the scientific community and established the reputations of both Hamilton and Lloyd.

From 1833 onward, Hamilton adapted his optical methods to the study of problems in dynamics. Out of laborious preparatory work emerged an elegant theory, associating a characteristic function with any system of attracting or repelling point particles. If the form of this function is known, then the solutions of the equations of motion of the system can easily be obtained. Hamilton's two major papers "On a General Method in Dynamics" were published in 1834 and 1835. In the second of these, the equations of motion of a dynamical system are expressed in a particularly elegant form (Hamilton's equations of motion). Hamilton's approach was further refined by the German mathematician Carl Jacobi, and its significance became apparent in the development of celestial mechanics and quantum mechanics. Hamiltonian mechanics underlies contemporary mathematical research in symplectic geometry (a field of research in algebraic geometry) and the theory of dynamical systems.

In 1835 Hamilton was knighted by the lord lieutenant of Ireland in the course of a meeting in Dublin of the British Association for the Advancement of Science. Hamilton served as president of the Royal Irish Academy from 1837 to 1846.

Hamilton had a deep interest in the fundamental principles of algebra. His views on the nature of real numbers were set forth in a lengthy essay, "On Algebra as the Science of Pure Time." Complex numbers were then represented as "algebraic couples"—i.e., ordered pairs of real numbers, with appropriately defined algebraic operations. For many years Hamilton sought to construct a theory of triplets, analogous to the couplets of complex numbers, that would be applicable to the study of three-dimensional geometry. Then, on October 16, 1843, while walking with his wife beside the Royal Canal on his way to Dublin, Hamilton suddenly realized that the solution lay not in triplets but in

quadruplets, which could produce a noncommutative four-dimensional algebra, the algebra of quaternions. Thrilled by his inspiration, he stopped to carve the fundamental equations of this algebra on a stone of a bridge they were passing.

Hamilton devoted the last 22 years of his life to the development of the theory of quaternions and related systems. For him, quaternions were a natural tool for the investigation of problems in three-dimensional geometry. Many basic concepts and results in vector analysis have their origin in Hamilton's papers on quaternions. A substantial book, *Lectures on Quaternions*, was published in 1853, but it failed to achieve much influence among mathematicians and physicists. A longer treatment, *Elements of Quaternions*, remained unfinished at the time of his death.

In 1856 Hamilton investigated closed paths along the edges of a dodecahedron (one of the Platonic solids) that visit each vertex exactly once. In graph theory such paths are known today as Hamiltonian circuits.

David Wilkins