Diagrammatic representation of the optimal performance of an endoreversible Carnot engine at maximum power output

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Abstract. It is shown that for three different arrangements of heat resistances the standard optimum performance criteria of endoreversible Carnot engines at maximum power output can be conveniently expressed by two simple diagrams. It is also pointed out that such diagrams should rightfully be referred to as Bejan diagrams.

1. Introduction

Bejan first proposed some simple diagrams to represent the performance of reversible and irreversible Carnot cycles [1, 2]. Later Bucher [3] used a similar diagram to represent the heat and work flows, the efficiency, and other performance parameters of a reversible Carnot cycle and to assist detailed discussions. These diagrams show clearly and accurately both the first and second laws of thermodynamics in operation, whereas more traditional diagrams depict only the first law of thermodynamics. Thus the present diagrams are very useful in teaching thermodynamics. As a result such diagrams have been repeatedly extended to wider classes of problems [4–10]. In particular, Wallingford [6] shows how irreversibilities distort the diagrams for heat engines and heat pumps. In this paper we build on these constructions and develop two simple diagrams by which the optimum performance of Carnot heat engines at maximum power output can be expressed directly.

2. Diagrammatic expressions

We consider an endoreversible Carnot engine with heat conductances k_1 and k_2 connecting it to its hot and cold reservoirs at temperatures T_h and T_c, respectively, as illustrated in figure 1. In order to derive the optimum performance of this engine at maximum power output we construct the diagrams shown in figure 2 (the detailed rules are given in the appendix). It should be emphasized that the presupposition behind the construction of figure 2 is that heat transfer obeys a linear Newtonian law. In figure 2(*b*) it is further assumed that the two heat resistances are identical, $k_1 = k_2$.

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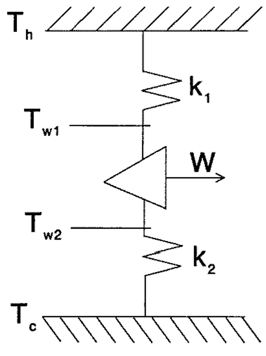
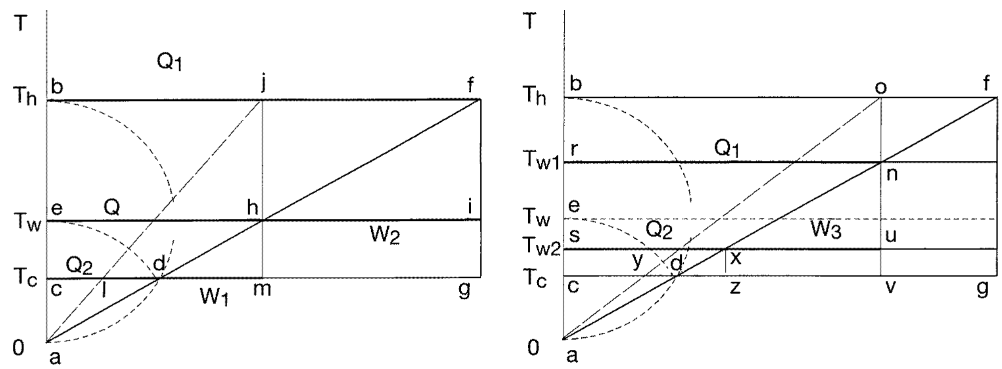


Figure 1. Schematic diagram of an endoreversible Carnot engine. See the text for definitions of variables.



(*a*) (*b*)

Figure 2. Diagrammatic description of the performance of endoreversible Carnot engines at maximum power output. (*a*) Engine with heat resistance only to the incoming heat current or to the outgoing heat current. (*b*) Engine with heat resistances to both the incoming and outgoing heat currents. See the text and the appendix for a detailed explanation.

From figure 2 one can easily prove geometrically that

$$T_{\mathrm{w}} = \mathrm{ae} = \mathrm{ad} = \sqrt{\mathrm{ab} \times \mathrm{ac}} = \sqrt{T_{\mathrm{h}} T_{\mathrm{c}}} \tag{1}$$

$$T_{\mathrm{w1}} = \mathrm{ar} = \tfrac{1}{2}(\mathrm{ab} + \mathrm{ae}) = \tfrac{1}{2}(T_{\mathrm{h}} + T_{\mathrm{w}}) = \tfrac{1}{2}\left(T_{\mathrm{h}} + \sqrt{T_{\mathrm{h}} T_{\mathrm{c}}}\right) \tag{2}$$

$$T_{\mathrm{w2}} = \mathrm{as} = \tfrac{1}{2}(\mathrm{ae} + \mathrm{ac}) = \tfrac{1}{2}(T_{\mathrm{w}} + T_{\mathrm{c}}) = \tfrac{1}{2}(\sqrt{T_{\mathrm{h}} T_{\mathrm{c}}} + T_{\mathrm{c}}). \tag{3}$$

Below it is shown how the optimum performance of an endoreversible Carnot engine at maximum power output is directly expressed by figure 2. We will discuss the performance of Carnot engines operating in three different situations: with heat resistance to the incoming heat current only; with heat resistance to the outgoing heat current only; with heat resistances to both the incoming and outgoing heat currents. The last case of course includes the two former cases, but since they are simpler they should facilitate understanding of the more general diagrams.

2.1. A Carnot engine with heat resistance to the incoming heat current only

Such a Carnot engine [11] is characterized by the heat conductance k_1 between the heat source at temperature T_{h} and the working fluid of the heat engine being finite, while the heat conductance k_2 between the working fluid and the heat sink at temperature T_{c} is infinite (i.e. no resistance). Thus the irreversibility of finite-rate heat transfer only appears in the absorbing heat process, so the entire cycle time t is spent on the absorbing heat process, while the duration of the heat rejection process may be neglected. The temperature T_w of the working fluid during heat absorption is different from that of the heat source and is determined [4] by equation (1), while the temperature of the working fluid during heat rejection is identical to that of the heat sink T_c.

The amount of heat Q absorbed from the heat source at temperature T_h and delivered to the engine at T_w and the heat Q_2 rejected to the heat sink at temperature T_c by the working fluid per cycle as well as the work output W_1 per cycle may be obtained from the geometry in figure 2(a) and equation (1) as

$$Q = \mathrm{eh} = k_1(T_h - T_w)t = k_1\left(T_h - \sqrt{T_c T_h}\right)t \tag{4}$$

$$Q = \mathrm{eh} = \mathrm{cm} = \mathrm{cd} + \mathrm{dm} = Q_2 + W_1. \tag{5}$$

If there were no heat resistance to the heat source, the work output per cycle would be $W_{max} = \mathrm{lm}$. The similar triangles 'dmh' and 'hea' in figure 2(a) easily yield the efficiency of the heat engine:

$$\eta = \frac{W_1}{Q} = \frac{\mathrm{dm}}{\mathrm{eh}} = \frac{\mathrm{ce}}{\mathrm{ae}} = 1 - \frac{\mathrm{ac}}{\mathrm{ae}} = 1 - \frac{T_c}{T_w} = 1 - \sqrt{\frac{T_c}{T_h}} = \eta_{CA} \tag{6}$$

and the second-law efficiency of the heat engine [5, 6, 12, 13]:

$$\begin{aligned}\varepsilon &= \frac{W_1}{W_{max}} = \frac{\mathrm{dm}}{\mathrm{lm}} = \frac{\mathrm{eh(mh/ae)}}{\mathrm{eh(cb/ab)}} = \frac{1 - \mathrm{ac/ae}}{1 - \mathrm{ac/ab}} \\ &= \frac{1 - \sqrt{T_c/T_h}}{1 - T_c/T_h} = \frac{1}{1 + \sqrt{T_c/T_h}} = \frac{\eta_{CA}}{\eta_c}\end{aligned} \tag{7}$$

where η_{CA} is the Curzon–Ahlborn efficiency of a Carnot engine at maximum power output [14–19], and η_c and W_{max} are the efficiency and the work output of a corresponding reversible Carnot engine with the same heat input. Then the relations between the work output W_1, the rejected heat Q_2 per cycle, and the heat Q are

$$W_1 = \mathrm{dm} = \mathrm{eh}\left(\frac{\mathrm{mh}}{\mathrm{ae}}\right) = \mathrm{eh}\left(1 - \frac{\mathrm{ac}}{\mathrm{ae}}\right) = Q\left(1 - \sqrt{\frac{T_c}{T_h}}\right) \tag{8}$$

and

$$Q_2 = \mathrm{cd} = \mathrm{eh}\left(1 - \frac{\mathrm{ac}}{\mathrm{ae}}\right) = Q\sqrt{\frac{T_c}{T_h}} \tag{9}$$

which can be obtained directly from figure 2(a). The resulting maximum power output, using equations (4) and (8), is

$$P_{max} = \frac{W_1}{t} = k_1\left(\sqrt{T_h} - \sqrt{T_c}\right)^2. \tag{10}$$

If T_c is taken to be the environmental temperature, the availability loss A_1 per cycle may also be expressed by the segment length 'ld' in figure 2(a). The relation between A_1 and Q is

$$\begin{aligned}A_1 &= W_{max} - W_1 = \mathrm{ld} = \mathrm{lm} - \mathrm{dm} = \mathrm{eh}\left(\frac{\mathrm{cb}}{\mathrm{ab}} - \frac{\mathrm{mh}}{\mathrm{ae}}\right) \\ &= Q\left(\sqrt{\frac{T_c}{T_h}} - \frac{T_c}{T_h}\right) = T_c Q\left(\frac{1}{T_w} - \frac{1}{T_h}\right) = T_c \Delta S_1\end{aligned} \tag{11}$$

making the rate of availability loss

$$R_{A_1} = \frac{A_1}{t} = k_1\sqrt{\frac{T_c}{T_h}}\left(\sqrt{T_h} - \sqrt{T_c}\right)^2 \tag{12}$$

where $\Delta S_1 = Q(1/T_w - 1/T_h)$ is the entropy production per cycle which results from the finite-rate heat transfer between the heat source at temperature T_h and the working fluid at temperature T_w.

2.2. A Carnot engine with heat resistance to the outgoing heat current only

Such a Carnot engine with heat resistance to the outgoing heat current only [11] is the 'mirror image' of the case treated in subsection 2.1 in that the irreversibility of finite-rate heat transfer only appears in the heat rejection process. Thus, the temperature of the working fluid during the heat absorption is identical with that of the heat source T_h, while the temperature T_w of the working fluid during heat rejection is different from that of the heat sink and is determined by equation (1).

The amount of heat Q_1 absorbed from the heat source and the heat Q rejected to the heat sink by the working fluid per cycle and the work output W_2 per cycle, similarly to (4) and (5), are given by

$$Q_1 = \mathrm{bf} = \mathrm{ei} = \mathrm{eh} + \mathrm{hi} = Q + W_2 \tag{13}$$

$$Q = \mathrm{eh} = k_2(T_w - T_c)t = k_2(\sqrt{T_h T_c} - T_c)t \tag{14}$$

as illustrated in figure 2(*a*). The corresponding work output per cycle without any heat resistance to the heat sink, $W_{max} = \mathrm{dg}$. The similar triangles 'hif' and 'fba' in figure 2(*a*) also quickly yield the efficiency of this heat engine:

$$\eta = \frac{W_2}{Q_1} = \frac{\mathrm{hi}}{\mathrm{bf}} = \frac{\mathrm{if}}{\mathrm{ab}} = 1 - \frac{\mathrm{ae}}{\mathrm{ab}} = 1 - \sqrt{\frac{T_c}{T_h}} = \eta_{CA} \tag{15}$$

and the second-law efficiency of the heat engine:

$$\varepsilon = \frac{W_2}{W_{max}} = \frac{\mathrm{hi}}{\mathrm{dg}} = \frac{\mathrm{if}}{\mathrm{gf}} = \frac{\mathrm{ab} - \mathrm{ae}}{\mathrm{ab} - \mathrm{ac}} = \frac{T_h - \sqrt{T_h T_c}}{T_h - T_c} = \frac{\eta_{CA}}{\eta_c}. \tag{16}$$

Likewise the relations between the work output W_2, the heat input Q_1, the availability loss A_2 per cycle, and finally the heat rejected Q are

$$W_2 = \mathrm{hi} = \mathrm{eh}\left(\frac{\mathrm{if}}{\mathrm{ae}}\right) = Q\left(\sqrt{\frac{T_h}{T_c}} - 1\right) \tag{17}$$

$$Q_1 = \mathrm{bf} = \mathrm{eh}\left(\frac{\mathrm{ab}}{\mathrm{ae}}\right) = Q\sqrt{\frac{T_h}{T_c}} \tag{18}$$

$$\begin{aligned} A_2 &= W_{max} - W_2 = \mathrm{dm} = \mathrm{eh}\left(\frac{\mathrm{mh}}{\mathrm{ae}}\right) \\ &= Q\left(1 - \sqrt{\frac{T_c}{T_h}}\right) = T_c Q\left(\frac{1}{T_c} - \frac{1}{T_w}\right) = T_c \Delta S_2 \end{aligned} \tag{19}$$

where $\Delta S_2 = Q(1/T_c - 1/T_w)$ is the entropy production per cycle which results from the finite-rate heat transfer between the working fluid at temperature T_w and the heat sink at temperature T_c. Thus the maximum power output and the rate of availability loss may be expressed as

$$P_{max} = \frac{W_2}{t} = k_2\left(\sqrt{T_h} - \sqrt{T_c}\right)^2 \tag{20}$$

$$R_{A_2} = \frac{A_2}{t} = k_2\sqrt{\frac{T_c}{T_h}}\left(\sqrt{T_h} - \sqrt{T_c}\right)^2. \tag{21}$$

It is important to note that when $k_1 = k_2$, the two heat engines represented by figure 2(*a*) not only have the same efficiency and second-law efficiency, but also the same maximum power output and rate of availability loss, although they have different heat inputs and work outputs.

2.3. A Carnot engine with heat resistances to both the incoming and outgoing heat currents

In this engine both heat conductances k_1 and k_2 between the working fluid of the heat engine and the two heat reservoirs at temperatures T_h and T_c are finite. The irreversibility of finite-rate heat transfer appears in both heat exchange processes, and the times spent on these are t_1 and t_2, respectively. The temperatures $T_{\mathrm{w}1}$ and $T_{\mathrm{w}2}$ of the working fluid during the heat exchange processes are different from those of the heat source and the heat sink. Please recall that $k_1 = k_2 = k$ is assumed for constructing figure 2(*b*). Under this assumption, $T_{\mathrm{w}1}$ and $T_{\mathrm{w}2}$ may be determined directly by equations (2) and (3). Thus, the heats Q_1 absorbed from the heat source and Q_2 rejected to the heat sink by the working fluid per cycle and the work output W_3 per cycle are given by

$$Q_1 = \mathrm{rn} = k(T_\mathrm{h} - T_{\mathrm{w}1})t_1 = \tfrac{1}{2}k\big(T_\mathrm{h} - \sqrt{T_\mathrm{h}T_\mathrm{c}}\big)t_1 \tag{22}$$

$$Q_2 = \mathrm{sx} = k(T_{\mathrm{w}2} - T_\mathrm{c})t_2 = \tfrac{1}{2}k\big(\sqrt{T_\mathrm{h}T_\mathrm{c}} - T_\mathrm{c}\big)t_2 \tag{23}$$

$$Q_1 = \mathrm{rn} = \mathrm{su} = \mathrm{sx} + \mathrm{xu} = Q_2 + W_3 \tag{24}$$

as determined from figure 2(*b*). Without any heat resistances the maximum work produced per cycle would be $W_\mathrm{max} = \mathrm{yv}$. The similar triangles ‘xun’ and ‘nra’ in figure 2(*b*) indicate the efficiency of the heat engine directly:

$$\begin{aligned}\eta &= \frac{W_3}{Q_1} = \frac{\mathrm{xu}}{\mathrm{rn}} = \frac{\mathrm{sr}}{\mathrm{ar}} = 1 - \frac{\mathrm{as}}{\mathrm{ar}} = 1 - \frac{T_{\mathrm{w}2}}{T_{\mathrm{w}1}} = 1 - \sqrt{\frac{T_\mathrm{c}}{T_\mathrm{h}}}\\ &= \eta_\mathrm{CA} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{t_2}{t_1}\sqrt{\frac{T_\mathrm{c}}{T_\mathrm{h}}}\end{aligned} \tag{25}$$

and the second-law efficiency of the heat engine:

$$\varepsilon = \frac{W_3}{W_\mathrm{max}} = \frac{\mathrm{xu}}{\mathrm{yv}} = \frac{\mathrm{sr/ar}}{\mathrm{cb/ab}} = \frac{1 - \mathrm{as/ar}}{1 - \mathrm{ac/ab}} = \frac{1 - T_{\mathrm{w}2}/T_{\mathrm{w}1}}{1 - T_\mathrm{c}/T_\mathrm{h}} = \frac{\eta_\mathrm{CA}}{\eta_\mathrm{c}}. \tag{26}$$

The corresponding work output is

$$\begin{aligned}W_3 &= \mathrm{xu} = \mathrm{rn}\left(\frac{\mathrm{un}}{\mathrm{ar}}\right) = \mathrm{rn}\left(1 - \frac{\mathrm{as}}{\mathrm{ar}}\right) = Q_1\left(1 - \frac{T_{\mathrm{w}2}}{T_{\mathrm{w}1}}\right)\\ &= Q_1\left(1 - \sqrt{\frac{T_\mathrm{c}}{T_\mathrm{h}}}\right)\end{aligned} \tag{27}$$

and the availability loss is

$$\begin{aligned}A_3 &= W_\mathrm{max} - W_3 = \mathrm{yz} = \mathrm{yd} + \mathrm{dz} = \mathrm{yv} - \mathrm{dv} + \mathrm{dz}\\ &= \mathrm{rn}\left(\frac{\mathrm{cb}}{\mathrm{ab}} - \frac{\mathrm{cr}}{\mathrm{ar}}\right) + \mathrm{sx}\left(\frac{\mathrm{cs}}{\mathrm{as}}\right)\\ &= \mathrm{ac}\left[\mathrm{rn}\left(\frac{1}{\mathrm{ar}} - \frac{1}{\mathrm{ab}}\right)\right] + \mathrm{sx}\left(1 - \frac{\mathrm{ac}}{\mathrm{as}}\right)\end{aligned}$$

$$= T_c Q_1\left(\frac{1}{T_{w1}} - \frac{1}{T_h}\right) + T_c Q_2\left(\frac{1}{T_c} - \frac{1}{T_{w2}}\right)$$

$$= T_c(\Delta S_3 + \Delta S_4) = Q_1\left(\sqrt{\frac{T_c}{T_h}} - \frac{T_c}{T_h}\right) \tag{28}$$

where $\Delta S_3 = Q_1(1/T_{w1} - 1/T_h)$ and $\Delta S_4 = Q_2(1/T_c - 1/T_{w2})$ are, respectively, the entropy productions of the two heat transfer processes. The line segments 'yd' and 'dz' in figure 2(*b*) directly represent the availability losses of the two heat transfer processes. It should be emphasized that the relationships between availability loss and entropy production expressed by equations (11), (19), and (28) are general results of equilibrium thermodynamics as discussed by Tolman and Fine [19] and others [20, 21].

From equation (25) one obtains an important relation for dividing the cycle duration between the two heat transfer processes:

$$t_1 = t_2 = \tfrac{1}{2}t. \tag{29}$$

This shows clearly that two important relations for Carnot engines at maximum power output may be obtained directly from figure 2(*b*), namely the optimum ratio of the temperatures of the working fluid in the two heat exchange processes, equations (2) and (3), and the optimum ratio of the times spent on the two heat exchange processes, equation (29). Using equations (22) and (27)–(29) one further obtains the maximum power output

$$P_{max} = \frac{W_3}{t} = \frac{k}{4}\left(\sqrt{T_h} - \sqrt{T_c}\right)^2 \tag{30}$$

and the rate of availability loss

$$R_{A3} = \frac{A_3}{t} = \frac{k}{4}\sqrt{\frac{T_c}{T_h}}\left(\sqrt{T_h} - \sqrt{T_c}\right)^2. \tag{31}$$

3. Discussion

When the heat conductances between the working fluid of a Carnot engine and its two heat reservoirs are different, the optimum performance of the Carnot engine at maximum power output can also be expressed by a diagram. However, it is then necessary to introduce an equivalent temperature [8]

$$T_w = T_h - \frac{Q_1}{t}\frac{\left(\sqrt{k_1} + \sqrt{k_2}\right)^2}{k_1 k_2}. \tag{32}$$

The efficiency at maximum power output is still η_{CA}, and the maximum power output is now given [8, 22, 23] by

$$P_{max} = \frac{k_1 k_2}{\left(\sqrt{k_1} + \sqrt{k_2}\right)^2}\left(\sqrt{T_h} - \sqrt{T_c}\right)^2. \tag{33}$$

It is interesting to note that the three heat engines mentioned above are the special cases of $k_2 \to \infty$, $k_1 \to \infty$ and $k_1 = k_2$, and equations (10), (20) and (30) can respectively be derived directly from equation (33) in these limits.

As mentioned above, the three heat engines at maximum power output have the same efficiency η and second-law efficiency ε. However, their work outputs and availability losses are different because their cycle times and heat inputs per cycle are different.

From a historical point of view, such diagrams [1–10], which can clearly show the performance of typical thermodynamic cycles, should properly be referred to as Bejan diagrams since the fundamental diagram was first proposed by Bejan [1] nine years before Bucher [3].

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Appendix

Figures 2(*a*) and 2(*b*) may easily be constructed by using the following rules:

1. Draw a vertical axis of absolute temperatures T with $T = 0$ at the point 'a'.
2. Mark the points 'b' and 'c' on the T axis at the temperatures T_h and T_c of the heat source and heat sink between which the Carnot engine operates and draw horizontal lines through these points.
3. Draw a semicircle having the segment 'ab' as its diameter. The point of intersection between the semicircle and the horizontal line through 'c' is labelled 'd'.
4. Draw a slanted line through 'a' and 'd'. The point of intersection of this line with the horizontal line through 'b' is labelled 'f'.
5. Draw a vertical line through 'f'. The point of intersection of this line with the horizontal line through 'c' is labelled 'g'.
6. Mark a point 'e' on the T axis at a temperature T_w such that the distance 'ae' equals the length of the segment 'ad'.
7. In figure 2(*a*) the horizontal line through 'e' intersects the slanted line 'af' at the point 'h' and the vertical line 'fg' at the point 'i'. A vertical line drawn through 'h' intersects the horizontal line 'cg' at the point 'm' and the horizontal line 'bf' at the point 'j'.
8. Draw a slanted (dashed) line from 'a' to 'j'. This line intersects the horizontal line 'cg' at the point 'l'.
9. The lengths of the segments 'bf', 'eh', 'cd', 'dm' and 'hi' represent the heat and work exchanges Q_1, Q, Q_2, W_1 and W_2, respectively, defined in subsections 2.1 and 2.2.

6′. In figure 2(*b*) steps 1 to 5 are the same as above. Next, mark the midpoints 'r' and 's' of the segments 'be' and 'ec' and draw horizontal lines through 'r' and 's'. The corresponding temperatures are T_{w1} and T_{w2}.

7′. The two horizontal lines through 'r' and 's' intersect the slanted line 'af' at the points 'n' and 'x', respectively. A vertical line drawn through 'n' intersects the horizontal lines through 'b', 's', and 'c' at the points 'o', 'u', and 'v', respectively.

8′. Draw a slanted (dashed) line from 'a' to 'o'. This line intersects the horizontal line 'cg' at the point 'y'.

9′. The lengths of the segments 'rn', 'sx' and 'xu' represent the heat and work exchanges Q_1, Q_2 and W_3, respectively (see subsection 2.3).

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