OPTIMAL CONTROL CONFIGURATION OF HEATING AND HUMIDIFICATION PROCESSES part I

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In prezenta lucrare se iau in considerare procesele fundamentale de transfer de caldura si control al umiditatii iar procesul politropic de incalzire a aerului cu modificarea umiditatii este analizat prin prisma teoriei optimizarii. Scopul este de a gasi metode optime de a incalzi si umidifica aerul producind entropie minima.

Am considerat interactiunea in contra-curent dintre aer exterior rece si apa calda aduse in contact intr-un echipament special destinat umidificarii. Acest studiu se bazeaza pe teoria controlului optimal iar scopul sau este gasirea parametrilor optimi pe baza generarii minime de entropie asociate procesului.Folosim teoria optimizarii pentru a analiza modul in care prin varierea suprafatei de contact dintre aer si apa se obtin temperatura si umiditatea dorite pentru aer la iesirea din echipamentul de umidificare.Temperatura apei si cantitatea de apa optime la intrarea in umidificator sint determinate.

The fundamental heat transfer and humidity control processes are considered and the polytropic process of heating air while modifying the humidity contents is analyzed using the optimal control theory. The aim is to investigate optimal ways of heating and humidifying the air while producing minimum entropy.

Heating and humidification are considered here in a counter current interaction between dry, cold outdoor air and hot water in a device purposely used for bringing air in contact with water. This study is based on the theory of optimal control and its aim is to find the optimal parameters based on the minimum entropy production associated to the process. We use optimal control theory to analyze how varying the contact surface between water and air achieves the desired temperature and humidity of the air at the exit of the humidifying device. The optimal water temperature and amount at the inlet are also determined.

Keywords: optimal control theory, minimum entropy production, air humidification, heating

Nomenclature

Latin symbols

A = area of water surface in contact with air; B = atmospheric pressure

$C_1 \ldots C_5$ = empirical coefficients; C_v = heat capacity

d = number of controls; $\vec{d}$ = vector of controls for the process

$\vec{F}$ = vector of evolution expressions for $\vec{n}$

$G(\vec{n}_1, t_1)$ = desired combination of state functions at the final time

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G = vaporization rate of water; H = Hamiltonian of optimization
ΔH = enthalpy of water vapor; I = cost (object) functional for the process
k = coefficient of heat transfer; L = rate of dissipation
M = molar mass; m_v = mass of water vapor per volume of air
m_w = mass of water in the water layer per area of surface of water-air contact
n = number of state functions; $\vec{n}$ = vector of state functions of the system
p = partial pressure of water; Q = rate of heat transfer
R = ideal gas constant
ΔS = entropy change (production) during the process
T = temperature; t = time; v = air velocity over water surface
V = volume; x = position in flow direction
y = width of channel; z = height of air channel

Superscripts
* = optimal value $\dot{s} = ds/dt$

Subscripts
$_0$ = initial value; $_1$ = final value; $_a$ = related to air
$_{sat}$ = at saturation; $_v$ = related to vapor; $_w$ = related to water

Greek symbols
λ_l = latent heat of water; $\vec{\psi}$ = vector of co-state (adjoint) variables
$\vec{\omega}$ = vector of state variables T_a, T_w, m_v, m_w; φ = relative humidity of air

1. Introduction

In this paper we undertake an analysis of the process of heating and humidification of moist air using optimal control theory as the tool of the analysis. Moist air is brought into direct contact with water thereby increasing its humidity content while at the same time modifying the temperature.

The aim is to underpin the principles that can be used for air treatment so that both humidification and heating can be achieved at the same time.

The intent is to treat the thermodynamic variables describing the states of the moist air and the water as dynamic variables. Such dynamic evolution constraints are most easily incorporated into an optimal control formulation of the problem as opposed to calculus of variations. The fluids are described as ideal with the air characterized by 2 explicitly independent variables, temperature and humidity contents (expressed as water vapor mass per volume of air), while the water is characterized by its temperature and mass.

The current HVAC (Heating, Ventilating and Air Conditioning) literature mentions the following theoretical psychrometric processes[1-5]:

    a) isothermal humidification – in which saturated steam is sprayed into the air stream. In theory it is considered that it evolves as an isothermal

process with very little influence of the steam temperature on the temperature of the air at the exit from the humidification device

b) adiabatic humidification - in which hot water is sprayed into the air stream. In theory it is considered that it evolves as an adiabatic process in which the hot water used for humidification does not modify its temperature.

In this paper we approached the real process of air treatment with water in view of achieving not only the desired degree of relative humidity of the air at the exit from the humidification device but also the air heating.

Note that we set to analyze the real process and that the transfer of mass and energy occur through evaporation.

To our knowledge this is the first time when optimal control theory is used to analyze the process of both air humidification and heating. The purpose is a theoretical investigation into the possibility of treating moist air in HVAC equipment while saving energy.

There has been little investigation in the psychrometric literature regarding the effects of the water temperature on the humidification process so that either heating or cooling can be achieved in the same time by means of adjusting the relative humidity of the air. Even for the evaporative cooling process – well documented otherwise – the temperature of the water for humidification is not mentioned.

The optimal control theory has in exchange been used in numerous thermodynamic applications [6-7] for heat engines [8-10]; heat exchangers [11], [12]; gas turbines [13]. Other related thermodynamic applications were developed in [14].

2. Elements of optimal control theory

The principles of the applications of the optimal control theory to thermodynamic systems are presented is detail in [15].

The state of a thermodynamic system is at any time characterized by a set of n independent real numbers which may be visualized as a vector $\vec{n}$ in an n – dimensional Euclidian vector space.

The system has d controls whose values influence the evolution of the system state. The controls can be visualized as a vector $\vec{d}$ in a d – dimensional Euclidian vector space. Usually, the admissible controls are limited to a region of this space.

Systems of the type analyzed here are dynamical systems meaning that their state at any time $t > t_0$ with t_0 being the initial time, is uniquely determined by a set of differential equations

$$\dot{\vec{n}}(t) = \vec{F}[n(t), \vec{d}(t)] \qquad (2.1.)$$

and boundary conditions $\vec{n}(t) \leq \vec{n}(t_0)$

The inequality sign is used in the sense that the vector elements are compared one by one.

Here $\vec{F}[\vec{n}, \vec{d}\ ]$ and $\dfrac{\partial \vec{F}[\vec{n},d\ ]}{\partial n_j}$ are piecewise continuous vector functions of $\vec{n}$ and $\vec{d}$, and $\vec{d}(t)$ is a piecewise continuous vector function that is an admissible control.

The analyzed system is autonomous ($\vec{F}$ does not depend explicitly on time) and does not have memory – which means that the evolution of the system for $t > t_1\ (> t_0)$ depends only on $\vec{n}(t_1)$ and $\vec{d}(t)$ for $t > t_1$. The controls are non-inertial, that is they may change discontinuously.

A cost functional (an index of the performance) needs to be defined, that specifies the operating goal of the system:

$$I = G(\vec{n}_1, t_1) + \int_{t_0}^{t_1} L[\,n(t), \vec{d}(t)]\ ,\qquad\qquad (2.2.)$$

where $\vec{n}_1 = \vec{n}(t_1)$ is the final state of the system, $G(\vec{n}_1, t_1)$ is the value of one of the parameters at the end of the process, and $L[n(t), \vec{d}(t)]$ the rate of dissipation. The chosen objective function for our case is $I = -\Delta S$, i.e. we wish to minimize the entropy production of the process, as used for cryogenic systems [16]. The final state of the process is not specified. The optimal control solution requires us to find an admissible control $\vec{d}^*(t)$ such that the system is driven from its initial state to the final state in such a manner that the performance index is maximized.

This control will be the optimal control, and the phase space trajectory $\vec{n}^*(t)$ will be the optimal trajectory. The initial state of the system differs from the final state in our case.

The Pontryagin maximum principle states a set of necessary conditions for solving the optimum control problem (the minimum conditions to be met).
First the Hamiltonian is defined as

$$H(\vec{n}, \vec{d},\vec{\psi}) \equiv \vec{L}[\vec{n}, \vec{d}\ ] + \vec{\psi}\ \vec{F}(\vec{n}, \vec{d}\ )\qquad\qquad (2.3.)$$

The vector function $\vec{\psi}(t)$ is a co-state variable or adjoint variable. Its role is to ensure that the equation (3.1.) is satisfied.
It satisfies an equation of motion:

$$\dot{\vec{\psi}}(t) = -\dfrac{\partial H}{\partial \vec{n}}\,[\vec{n}(t),\, \vec{d}(t),\vec{\psi}(t)]\qquad\qquad (2.4.)$$

where $\left(\dfrac{\partial H}{\partial \vec{n}}\right)_j = \dfrac{\partial H}{\partial n_j}$

The function H being a Hamiltonian, equation (3.1.) can be written as

$$\dot{\vec{n}}(t) = \dfrac{\partial H}{\partial \vec{\psi}}\,[\vec{n}(t),\, \vec{d}(t),\, \vec{\psi}(t)]\qquad\qquad (2.5)$$

The Pontryagin maximum principle states that if $\vec{d}^{*}(t)$ is an admissible control and $\vec{n}^{*}(t)$ is the trajectory corresponding to $\vec{d}^{*}$ which satisfies the boundary conditions

$\vec{n}^{*}(t_{0}) = \vec{n}_{0}$ and $\vec{n}^{*}(t_{1}) = \vec{n}_{1}$,

then for $\vec{d}^{*}(t)$ to be an optimal control it is necessary that $\vec{n}^{*}(t)$ and $\vec{\psi}^{*}(t)$ satisfy the canonical equations

$$\dot{\vec{n}}^{*}(t) = \frac{\partial H}{\partial \vec{\psi}}\,[\vec{n}^{*}(t),\, \vec{d}^{*}(t), \vec{\psi}^{*}(t)] \tag{2.6}$$

$$\dot{\vec{\psi}}^{*}(t) = -\frac{\partial H}{\partial \vec{n}}\,[\vec{n}^{*}(t),\, \vec{d}^{*}(t),\, \vec{\psi}^{*}(t)] \tag{2.7}$$

with $\vec{n}^{*}(t_{0}) = \vec{n}_{0}$ and $\vec{n}^{*}(t_{1}) = \vec{n}_{1}$.

Moreover, the function $H\,[\vec{n}^{*}(t),\, \vec{d}^{*}(t),\, \vec{\psi}^{*}(t)]$ is an absolute maximum over the set of admissible controls for t in $[t_{0},\, t_{1}]$ meaning that

$$H\,[\vec{n}^{*}(t),\, \vec{d}^{*}(t),\, \vec{\psi}^{*}(t)] \geq H\,[\vec{n}^{*}(t),\, \vec{d}(t),\, \vec{\psi}^{*}(t)] \tag{2.8}$$

for any admissible $\vec{d}$.

$$H^{*} \equiv H\,[\vec{n}^{*}(t),\, \vec{d}^{*}(t),\, \vec{\psi}^{*}(t)] \tag{2.9}$$

is a constant for an autonomous system.

Equation (2.9) represents energy conservation for dynamic systems which are invariant under time translation. This equation can be generalized for non-autonomous systems to: $\dfrac{dH}{dt} = \dfrac{\partial H}{\partial t}$.

The Pontryagin maximum principle provides the necessary conditions for finding a solution to the optimal control problem. If there is a solution to equations (2.6) and (2.7) which satisfies the boundary conditions (2.8) and (2.9) then it is an optimal solution.

The optimum solution has the following property:

Since $\vec{d}(t)$ is piecewise continous, $\vec{n}(t)$, $\vec{\psi}(t)$ are continous as they are solutions to the equations (2.6.) and (2.7.)

According to the principle of optimality, any portion of the optimum trajectory is also an optimal trajectory. That means, if $\vec{n}^{*}(t)$ is an optimum trajectory starting at $\vec{n}^{*}(t_{0}) = \vec{n}_{0}$ and ending at $\vec{n}^{*}(t_{1}) = \vec{n}_{1}$ and $\vec{d}^{*}(t)$ is the optimum control on $[t_{0}, t_{1}]$ corresponding to that trajectory, then if an optimum trajectory $\hat{n}^{*}(t)$ is sought on the interval $[\hat{t}_{0}, t_{1}]$ where $t_{0} < \hat{t}_{0} < t_{1}$ such that $\hat{\vec{n}}^{*}(\hat{t}_{0}) = \vec{n}_{0}$ and $\hat{\vec{n}}^{*}(\hat{t}_{1}) = \vec{n}_{1}$, that trajectory $\hat{\vec{n}}^{*}(\hat{t}) = \vec{n}^{*}(t)$ is the optimum trajectory and $\hat{\vec{d}}^{*}(t) = \vec{d}^{*}(t)$ is the optimum control.

3. Mathematical model of the humidification process

The assumptions for modeling the process are:

- Moist air is blown into a humidification channel over a stream of water; we consider the case that the water temperature is higher than the air temperature, so humidification is accompanied alongside with heating the air
- Both water velocity and air velocity are taken into account and we assume counterflow
- The channel has a given finite length, the same height throughout its length, while its witdh is variable
- The temperatures of water and air depend on the position along the channel but not the perpendicular directions, i.e. we assume perfect mixing across the flow front.

The process that is going to be modeled is a non-equilibrium thermodynamic process. We are interested in finding the optimum solution for the transfer of heat and mass between water and air. The variables that are analyzed are the air temperature and humidity content (expressed as mass of water vapor per volume of air) as air is blown into the humidification channel and the water inlet temperature and flowrate.

The initial state of the system differs from the final state as we are interested in meeting certain conditions for the air being treated (humidified and heated), in terms of temperature and relative humidity. The parameters (the state variables) for the air which is treated are set for the initial state (as boundary conditions).

The air stream and the water layer will exchange heat and mass as water will evaporate into the air.

In order to solve the optimum control problem, the state variables are defined as follows:

1. $T_a(x)$ = the air temperature,   [K] (state variable for the air side)

2. $T_w(x)$ = the water layer temperature,   [K]   (state variable for the water side)

3. $m_v(x)$ = mass of water vapor per volume of air   [kg/m^3] (state variable for the air side)

4. $m_w(x)$ = mass of water in the water layer per   [kg/m^2] (state variable area of surface of water-air contact   for the water side)

Note the way the last two state variables are set: per volume of air - $m_v(x)$ - and per area of surface in contact - $m_w(x)$. The intent for them is to have the form of a water vapor density and a water layer density per area respectively.

The system is considered in steady-state is autonomous and does not have memory.

The state variables are indicated as x-dependant, but as a first step the set of partial derivatives for the state variables will be shown as time dependant for a more "classical" approach.

Then the motion equations of the above defined state variables are presented as dependant on the position along the humidification channel, and the Hamiltonian are written for that set of equations.

We consider the temperatures of the air and water layer to be uniform throughout the two media at each value of x.

3.1. Constraints
3.1.1. Heat Transfer

First the heat exchanged is considered a function of the temperatures of the water and air, and depending on the width y of the channel as well [17].

$$Q = kA(T_w - T_a) \qquad [W] \qquad (3.1.1)$$

where k = coefficient of heat transfer

    A = area of water surface in contact with air

    T_w = water layer temperature; T_a = air temperature

Equation (3.1.1) is written according to the formula for heat transfer through convection between the two media (Newton's cooling law) and to the formula of heat transfer through conduction (Fourier's law).

3.1.2. Evaporation Rate

The rate at which water is transferred from the water stream into the moist air is [18].

$$G = C_1 A(1 - \varphi)(1 + C_2 v) \qquad [kg/s] \qquad (3.1.2)$$

Where G = evaporation rate of water

    A = area of water surface in contact with air

    C_1 = coefficient of the evaporation rate

    C_2 = coefficient of the dependency on air velocity v

    v = air velocity over water surface ; φ = relative humidity of air

A number of known equations were used and compiled to obtain equation (3.1.2).

First we had a look at the Clapeyron-Claussius equation for the change of phase – the water that vaporizes into the air.

$$\frac{dP}{dT} = \frac{\lambda_l}{T\Delta V} \qquad (3.1.3)$$

Then we considered Dalton's formula for large volumes of water

$$G = ACv \, (p_s - p_v) 1.013/B \qquad (3.1.4)$$

with B = atmospheric pressure

Experimental equations that involve evaporation of water have been considered as well [12].

To take into account the contributions of various constants present in the described formulae we used coefficients for the purpose of generalizing our equation.

3.1.3.   Time Dependent Constraints

So far we have the following set of constraints where the dot denotes time derivatives:

Energy flux on the air side - equivalent of equation (3.1.1)

1. $\dot{T_a} = \dfrac{k}{C_{va}} y (T_w - T_a)$ (3.1.9)

Energy flux on the water side – involving terms of heat transfer and heat of evaporation

2. $\dot{T_w} = - \dfrac{k}{C_{vw}} (T_w - T_a) - \dfrac{\lambda_l}{C_{vw}} A (1 - \dfrac{m_v R T_a}{V M_w C_2 e^{-\lambda_l/RT_a}})(1 + C_2 v) C_3 e^{-\lambda_l/RT_w}$ (3.1.10)

Mass conservation of water vapor in the air

3. $\dot{m_v} = - \dfrac{m_w}{z} = \dfrac{G}{Az}$ (3.1.11)

with z = vapor height

Mass conservation of water in the water layer – from equation (3.1.2)

4. $\dot{m_w} = - \dfrac{G}{A} = - (1 - \dfrac{\dfrac{m_v R/M_v T_a}{V}}{C_3 e^{-\lambda_l/RT_a}})(1 + C_2 v)$ (3.1.12)

where Q = heat transferred from water to air
k = heat conductivity; y = channel width
A = area of contact between air and water
C_1, C_2 = constants ; φ = relative humidity of air
G = evaporation rate of water; m_v = mass of water vapor in the air
m_w = mass of water for humidification
λ_l = latent heat of water; ΔH = enthalpy of water vapor

Below is presented a scheme of the analyzed process: The humidification channel with the water layer over which the air is flowing.
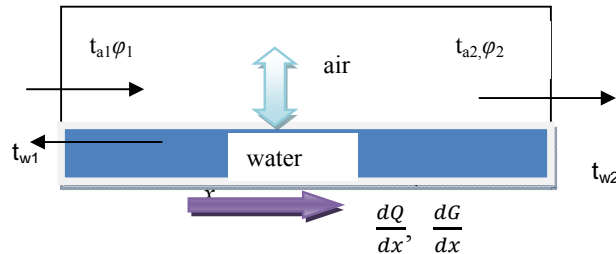
Fig. 3.1. Scheme of the humidification process – longitudinal section

Next, we derive the constraints in terms of the distance x along the channel.

3.2. *Energy conservation for the air flow through the channel*

An airflow will enter the channel (humidification coil) with the internal energy state described by the temperature T_{a1} in the state 1 and will leave then at the state 2 having the temperature T_{a2}.

The energy flux in the air at the states 1and 2 can be written:

$$Q_1 = C_{p,vol} \cdot v \cdot z \cdot y \, T_{a1} \tag{3.2.1}$$

$$Q_2 = C_{p,vol} \cdot v \cdot z \cdot y \, T_{a2} \tag{3.2.2}$$

where $C_{p,vol}$ = heat capacity of air per unit of volume at constant pressure
and $v \cdot z \cdot y$ = air volume flowrate through the humidification channel

The energy flow exchanged by the water and the air can be expressed as the conduction heat between the two media (water and air), for the differential area $y\Delta x$:

$$Q_c = ky\Delta x(T_w - T_a) \tag{3.2.3}$$

with k = heat conductivity of the pair water-air.

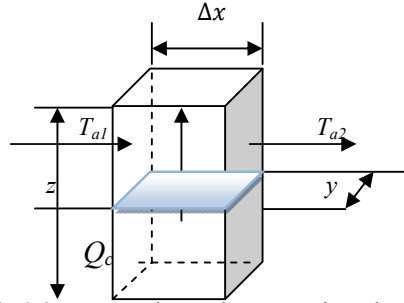Below, see a simple diagram for the heat exchange on the air side (Fig. 3.2)

Fig.3.2. Heat exchange between air and water

The figure above is illustrative for the equation of the heat exchanged between the air and the water through the surface of contact.

The energy conservation law reduces to:

$$Q_1 + Q_c = Q_2$$

$$C_{p,vol} \, v \cdot z \cdot y \, (T_{a1} - T_{a2}) + ky\Delta x(T_w - T_a) = 0 \tag{3.2.4}$$

$$\frac{dT_a}{dx} \approx \frac{\Delta T_a}{\Delta x} = \frac{k}{C_{p,vol} vz} (T_w - T_a) \tag{3.2.5}$$

with the constant C_4 defined by $v = \frac{C_4}{y}$ and v = air velocity through the channel.

Then equation (3.2.5) above becomes

$$\frac{dT_a}{dx} = \frac{k}{C_4 C_{p,vol} z} y \, (T_w - T_a) \tag{3.2.6}$$

3.3. *Water vapor mass conservation*

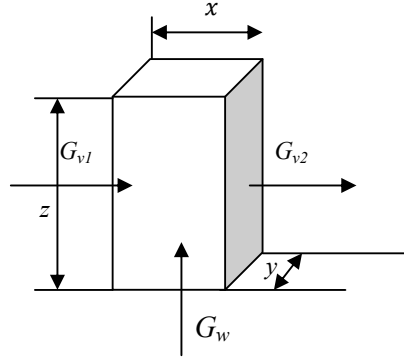Below, see a scheme for the mathematical model of mass conservation for the vapor on the air side:

Fig.3.3.Mass conservation of the water vapor

The water vapor flow into the volume considered above can be expressed as

$$G_{v1} = m_{v1} \, v \cdot z \cdot y \qquad\qquad\qquad (3.3.1)$$
$$G_{v2} = m_{v2} \, v \cdot z \cdot y \qquad\qquad\qquad (3.3.2)$$

The quantity of evaporated water into the air is

$$G_w = C_1 \, y \Delta x \, (1- \varphi)(\, 1 + C_2 v) \qquad\qquad (3.3.3)$$

the mass balance of water vapor may be written as

$$G_{v1} + G_w = G_{v2}$$

Thus, which becomes explicitly

$$v \cdot z \cdot y \, (m_{v2} - m_{v1}) \; = C_1 y \Delta x \, (1- \varphi)(\, 1 + C_2 \, v)$$

$$v = \frac{C_4}{y}$$

$$\frac{dm_v}{dx} \approx \frac{\Delta m_v}{\Delta x} = \frac{C_1 \, (1- \varphi)(\, 1 + C_2 \, v)}{v \, z} = C_1 \, (\frac{1-\varphi}{z})(\frac{y}{C_4} + C_2) \qquad (3.3.4)$$

From the definition of the state variable m_v (as a density of water vapor) it is possible to write

$$\frac{nR \, T_a}{V} = \frac{m_v \, R \, T_a}{M_w}$$

and substitute in the formula of the relative humidity p_v from the state equation:

$$\varphi(x) = \frac{p_v(x)}{p_{v,sat}(x)} = \frac{\frac{nR \, T_a}{V}}{C_3 e^{-\lambda_l/RT_a}} = \frac{m_v \, R \, T_a}{M_w C_3 e^{-\lambda_l/RT_a}} \qquad (3.3.5)$$

Then the differential equation for the state variable m_v becomes

$$\frac{dm_v}{dx} \approx \frac{\Delta m_v}{\Delta x} = \frac{C_1}{z} \, (1 - \frac{m_v \, R \, T_a}{M_w \, C_3 e^{-\lambda_l/RT_a}} \,) \, (\frac{y}{C_4} + C_2)$$

$$\frac{dm_v}{dx} = \frac{C_1}{z} \, (1 - \frac{m_v \, R \, T_a}{M_w \, C_3 e^{-\lambda_l/RT_a}} \,) \, (\frac{y}{C_4} + C_2). \qquad (3.3.6)$$

3.4. Energy conservation in the water

The energy content of the water in the two states in and out of the humidification channel can be written as

$$Q_{w1} = c_{p,w}\, v_w \cdot y \cdot m_w\, T_{w1} \tag{3.4.1}$$

$$Q_{w2} = c_{p,w}\, v_w \cdot y \cdot m_w\, T_{w2} \tag{3.4.2}$$

with $c_{p,w}$ = specific heat capacity of water at constant pressure [J/kgK].

Note again the way the state variable m_w was set initially, as mass of water for humidification per area of water-air contact surface, measured e.g. in [kg/m^2], thus allowing for the expressions (3.4.1 – 3.4.2) above.

The heat transferred through convection/conduction as seen from the water side is

$$Q_{\,w,c} = -ky\Delta x (T_w - T_a) \tag{3.4.3}$$

The heat transferred through evaporation is

$$Q_{\,w,e} = -\lambda_l G_w = -\lambda_l\, C_1 y \Delta x\, (1-\varphi)(\,1 + C_2\, v) \tag{3.4.4}$$

adding up to $Q_{\,w} = Q_{\,w,c} + Q_{\,w,e}$.

Energy conservation for the water can then be written as

$$Q_{w1} = Q_{\,w} + Q_{w2}$$

$$c_{p,w}\, v_w \cdot y \cdot m_w\, (T_{w2} - T_{w1}) - ky\Delta x (T_w - T_a) -$$

$$- \lambda_l C_1\, y\Delta x\, (1-\varphi)(\,1 + C_2\, v) = 0 \tag{3.4.5}$$

$$\frac{dT_w}{dx} \approx \frac{\Delta T_w}{\Delta x} = \frac{k(T_w - T_a) + \lambda_l C_1\,(1-\varphi)(\,1 + C_2 v)}{c_{p,w}\, v_w \cdot m_w} \tag{3.4.6}$$

$$v = \frac{C_4}{y} \quad \text{and} \quad v_w = \frac{C_5}{y}$$

$$\frac{dT_w}{dx} = \frac{yk(T_w - T_a) + \lambda_l C_1\,(1 - \dfrac{m_v\, R\, T_a}{M_w\, C_3 e^{-\lambda_l/RT_a}})(y + c_2\, c_4)}{c_{p,w}\, c_5 \cdot m_w} \tag{3.4.7}$$

3.5. Mass conservation for the water side

The flowrate of water into the humidification channel at the state 1 and 2 respectively are

$$G_{w1} = m_{w1}\, v_w \cdot y \tag{3.5.1}$$

$$G_{w2} = m_{w2}\, v_w \cdot y \tag{3.5.2}$$

The mass of water transferred through evaporation is

$$G_w = -\, C_1 y \Delta x\, (1-\varphi)(\,1 + C_2\, v) \tag{3.5.3}$$

Then mass conservation can be written as

$$G_{w2} + G_w = G_{w1} \tag{3.5.4}$$

Substituting (3.5.1), (3.5.2), and (3.5.3) into the equation (3.5.4), we obtain:

$$v_w \cdot y\, (m_{w2} - m_{w1}) - C_1 y \Delta x\, (1-\varphi)(\,1 + C_2\, v) = 0 \tag{3.5.5}$$

$$\frac{dm_w}{dx} \approx \frac{\Delta m_w}{\Delta x} = \frac{C_1(1-\varphi)(\,1 + C_2\, v)}{v_w} \tag{3.5.6}$$

$$v = \frac{C_4}{y} \quad \text{and} \quad v_w = \frac{C_5}{y}$$

$$\frac{dm_w}{dx} = \frac{C_1}{C_5}\,(1 - \frac{m_v\, R\, T_a}{M_w C_3 e^{-\lambda_l/RT_a}})(\,y + C_2\, C_4) \tag{3.5.7}$$

4. Resolution of the optimum control problem

Resolution of the problem implies finding solutions to the set of equations for the state variables among which there are the optimum solutions for the chosen control.

Let us summarize the canonical equations for the state variables as functions of x in order to compose the Hamiltonian:

We consider the vector space $\vec{\omega}$ of the state variables $T_a(x)$, $T_w(x)$, $m_v(x)$, $m_w(x)$ and form the Hamiltonian

$$H(\vec{\omega}, y, \vec{\psi}) = I(\vec{\omega}, y) + \vec{\psi} \cdot F(\vec{\omega}, y) \tag{4.1}$$

with $\vec{\psi}$ being the co-state vector so that

$$\frac{d\vec{\psi}}{dx} = -\frac{\partial H}{\partial \vec{\omega}}[\vec{\omega}(x), y(x), \vec{\psi}(x)] \tag{4.2}$$

$$\frac{d\vec{\omega}}{dx} = \frac{\partial H}{\partial \vec{\psi}}[\vec{\omega}(x), y(x), \vec{\psi}(x)] \ . \tag{4.3}$$

The equations for the co-state variables are given as

$$\frac{d\psi_1}{dx} = -\frac{\partial H}{\partial T_a} \tag{4.4}$$

$$\frac{d\psi_2}{dx} = -\frac{\partial H}{\partial T_w} \tag{4.5}$$

$$\frac{d\psi_3}{dx} = -\frac{\partial H}{\partial m_v} \tag{4.6}$$

$$\frac{d\psi_4}{dx} = -\frac{\partial H}{\partial m_w} \tag{4.7}$$

The process is steady-state, not cyclic.

We re-state the objective function which is I = minimum entropy generation as well as the control y = humidification coil width.

$$I = -\frac{ky(T_w - T_a)^2}{T_a T_w} \tag{4.8}$$

As described in section 2, we now apply Pontryagin's maximum principle to the set of equations above:

$$\Delta H = H[\vec{\omega}^*(x), y^*(x), \vec{\psi}^*(x)] - H[\vec{\omega}^*(x), y(x), \vec{\psi}^*(x)] \tag{4.9}$$

$\Delta H \geq 0$ for any allowed control $y(x)$.

$\Delta H \geq 0$ requires

$$(y^* - y)[\frac{k\psi_1}{C_4 C_{p,vol}\, z}(T_w - T_a) + \frac{k\psi_2(T_w - T_a)}{c_{p,w}\, c_5 m_w} - \frac{\lambda_l C_1 \psi_2 m_v R\, T_a e^{\lambda_l/RT_a}}{c_{p,w}\, c_5 m_w M_w\, C_3}$$

$$+\lambda_l C_1 \psi_2 + \frac{C_1 \psi_3}{z C_4} - \frac{C_1 \psi_3 m_v R\, T_a e^{\lambda_l/RT_a}}{z\, c_4 M_w\, C_3} + \frac{C_1 \psi_4}{C_5} - \frac{C_1 \psi_4 m_v R\, T_a e^{\lambda_l/RT_a}}{c_5 M_w\, C_3} -$$

$$- \frac{k(T_w - T_a)^2}{T_a T_w}] \geq 0 \tag{4.10}$$

whatever the choice of $y(x)$.

If the large bracket in expression (4.10) above is positive, y^* must assume its largest value allowed in order to assure a positive ΔH for any y. Correspondingly, if the bracket is negative, y^* must assume its smallest value assigned, typically 0, as a value smaller than 0 has no physical meaning for the channel width. These are boundary solutions, or bang-bang solutions. If finally the bracket equals zero over a distance x, y^* is undetermined by this argument. This is a so-called internal solution.

When the bracket vanishes over this interval, also all its derivatives must equal zero. These equations together provide the information necessary to find the solution for the optimal control y^* on this branch. Applying the Hamiltonian equations above indicates that the bracket is always positive i.e. the optimum solution to the present problem is the boundary solution $y^* = y_{max}$ over the entire length of the heater/humidifier.

The equations are solved with inlet/outlet boundary conditions on air temperature and humidity. The other parameters will follow in the solution from these parameters (as we will show in a future paper).

5. Conclusions

We investigated the interaction between water-air aiming to elucidate with the aid of the optimum control theory the way that the variation of the shape of contact surface between water and air leads to minimizing the destruction of availability in the process of mass and energy transfer between water and air.

Using optimum control theory it was possible to solve the system of 4 equations with 8 variables by fixing 4 of the variables at the boundaries and obtaining the remaining 4 from the optimization.

The result of solving the system being a bang-bang solution is a consequence of the control selection and the physical interpretation is that the minimum entropy is produced and the optimum path for the process of both heating and humidifying is achieved for a constant width of the humidifier/heater and that should be maximum value.

In practice, of course, the maximum value is limited by technical and economical feasibility.

In this instance, we investigated the process setting the width of the humidification device as a control function and minimum entropy production as an objective function. This is the first step in analyzing the mentioned process using the optimum control theory as the tool; for other objective, functions can be set – such as parameters of the air at the exit from the humidification device, parameters of the water at the inlet – or different control functions – such as water temperature or water consumption. While in some instances the investigation

results will match the intuitive ones (like in this case) some other might yield new interpretations.

In a future paper we will apply to the theoretical investigation numerical examples in order to have a better picture of various possibilities for applications in the engineering practice.

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