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Comment

Comment on "Essential discreteness in generalized thermostatistics with non-logarithmic entropy" by Abe Sumiyoshi

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This is an unusual comment. Acting as a referee for the paper in question [1], I had serious fundamental misgivings against which prof. Abe argued forcefully. The editor-in-chief of EPL thought that this fundamental disagreement would be of interest to readers of this journal and invited an open comment-reply. I accepted to forego my referee anonymity and have collected my concerns below. Let me also emphasize that I have no axe to grind in the long debate pro et con q-entropies (Tsallis entropies); I do not belong to any of those schools.

The sole stated reason for dismissing q-entropies for continuous systems in [1] is that such entropies would be "non-physical" which is later clarified to mean that they would contain Planck's constant h. What is physical or not is a matter of taste, except of course if the theory results in a contradiction to physical experiment. No such contradiction has been presented. However, one cannot axiomatically define what is physical. Further, it is very difficult to see why the presence of Planck's constant should be a rigid disqualification. We have many classical physical expressions which contain natural constants like the speed of light, the elementary charge, etc. Dismissing a continuous formulation of q-entropies solely for being "non-physical" is a self-created problem due to an overly restrictive definition of what is "physical". In the end experiment will answer that question.

The now classical information-theoretic expression for entropy,

$$S(\{p\}) = -\sum_{i=1}^{W} p_i \ln p_i$$

was indeed initially proven by Shannon for discrete systems [2], but soon after generalized to continuous systems [3]. The ensuing logarithmic divergence cancels when changes of entropy are considered. For non-extensive entropies like the q-entropies a similar argument does not remove the divergence, precisely because the entropies are not additive (eq. (10) of [1]). On the surface that is a problem, but digging a little deeper, this dependence on the size of the units considered is precisely the crux of why it has become necessary to define non-extensive entropies.

The fact that the full entropy of a particular system cannot be written solely as a sum of (tiny) subsystems is not in itself a violation of physical behavior. That point of view is only appropriate for macroscopic systems where nothing but bulk contributions to the entropy matter. In modern, much smaller systems, *e.g.*, the surface component of entropy becomes increasingly important, and those entropies are not (volume) additive. For the continuous limit of q-entropies this means that defining the measure $m(x)$ through a uniform parameter n going to infinity is inconsistent with the need for non-extensive quantities. The continuous limit must be made in the spirit of the q-concept, not the traditional additive way.

In summary, i) the appearance of a constant of Nature is not in itself "non-physical" except if it violates experiments; ii) the continuous limit of q-entropies must be made in accordance with its non-additivity. Consequently, [1] does not contain a proof that the continuous limit of q-entropies is outside reach of statistical mechanics, but rather represents a restricted view of what may be called entropy.

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REFERENCES

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[2] Shannon Claude E., *Bell Syst. Tech. J.*, **27** (1948) 379.
[3] Shannon Claude E., *Bell Syst. Tech. J.*, **27** (1948) 623.