# The Normal States of Magnetic Itinerant Electron Systems

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#### Abstract

The normal state of ferromagnetic d metals such as MnSi and ZrZn<sub>2</sub> with small or vanishing Curie temperatures may be described over a wide range in temperature and pressure in terms of a quantitative model of a marginal Fermi liquid based on dispersive spin fluctuations spectra inferred from inelastic neutron scattering data. The behaviour of antiferromagnetic f metals such as CePd<sub>2</sub>Si<sub>2</sub> and CeNi<sub>2</sub>Ge<sub>2</sub> with low or vanishing Néel Temperatures ( $T_N$ ) also appears unconventional, but the normal state above  $T_N$  has not yet been interpreted consistently in terms of an elementary extension of the spin-fluctuation model employed for the d-metal systems.

### 1 Introduction

The normal states of itinerant electron systems at low temperatures are normally described in terms of the Fermi liquid model. In perhaps the narrowest definition of this framework, the low-lying propagating modes of the interacting electron assembly are regarded as having a finite "overlap" with the non-interacting one-particle excitations (Anderson, 1995). This condition may be satisfied in the simple metals, but it has been called into question in particular for magnetic metals above small or vanishing Curie or Néel temperatures and more recently for the short-coherence-length superconductors (Anderson, 1995; Coleman, 1995a; Millis, 1993). In these strongly correlated electron systems, fluctuations of the order parameter may strongly suppress the transition temperature and give rise to a normal state with some unconventional properties.

Perhaps the simplest example of such behaviour may be found in pure ferromagnetic d metals such as MnSi and ZrZn<sub>2</sub> which have low Curie temperatures  $(T_C)$  that may be suppressed towards absolute zero with modest applied hydrostatic pressures. The temperature and pressure dependences of the resistivity and magnetic susceptibilities of these systems, together with the properties of the underlying spin fluctuation spectra, strongly suggest that the normal state may be usefully viewed in terms of a model of a marginal Fermi liquid (Moriya, 1985; Lonzarich, 1997). In this framework the usual fermion quasiparticle picture is retained, but the effective interaction between quasiparticles becomes long range and gives rise to low temperature behaviour not usually associated with the simplest Fermi liquid model.

In particular, the marginal Fermi liquid model relevant to this problem for  $T_C \rightarrow 0$  leads to a logarithmic divergence  $\ln(T^*/T)$  of the linear coefficient of the heat capacity C/T and a  $T^{-1/3}$  divergence in the quadratic coefficient of the resistivity  $\rho/T^2$  at low temperatures. This behaviour may be traced to a logarithmic divergence of the quasiparticle masses arising from the long-range quasiparticle interaction, and to a concomitant linear variation in the quasiparticle scattering rate as a function of energy or temperature near the Fermi surface. The divergence of the quasiparticle masses on the Fermi surface as  $T \rightarrow 0$  suggests a breakdown of the Fermi liquid state as defined at least in the narrow sense given above.

The marginal Fermi liquid represents, as the name implies, the weakest breakdown of the usual description of the normal metallic state. In more extreme cases, the starting picture of interacting fermion excitations on a conventional Fermi surface may itself have to be revised. This cannot be ruled out for some of the nearly magnetic or almost localised f-electron systems described below and the very short-coherence-length superconductors.

## 2 The quasiparticle–quasiparticle interaction

The breakdown of the simplest Fermi liquid description can be anticipated in some cases via an examination of the form of the quasiparticle–quasiparticle interaction. We may think of a quasiparticle excited near the Fermi level as interacting with various fields set up by other quasiparticles. Of particular interest, for an electron system near a ferromagnetic instability, is the exchange field essentially proportional to the local magnetisation, which couples to the spin moment of a quasiparticle. If we take this field acting on a given quasiparticle as a wave generated by another quasiparticle at some other point in space and time, we are led to an induced quasiparticle–quasiparticle interaction which is given, in the linear response approximation, by the space- and time-dependent magnetic susceptibility.

The spatial range of this interaction is then evidently the magnetic correlation length which diverges at the Curie temperature  $T_C$ . Thus, the quasiparticle interaction can become long range at low T as  $T_C \rightarrow 0$ . This leads to a singular scattering of quasiparticles near the Fermi surface which can qualitatively alter the character of the quasiparticle relaxation rate and, hence, of the low temperature properties in general.

For the standard model developed for the nearly ferromagnetic d metals, the qualitative temperature dependences of these properties depends chiefly on the dimension of space d (taken to be 3 for the cubic metals such as MnSi or ZrZn<sub>2</sub>) and the dynamical exponent z which characterises the propagation frequency spectrum for waves which carry the quasiparticle interaction (Millis, 1993; Moriya, 1985; Hertz, 1976; Lonzarich, 1997).

For our problem, this interaction is carried by magnetisation waves which tend to decay in time for  $T > T_C$ . Thus, the propagation spectrum is purely imaginary and characterised by the relaxation rate,  $\Gamma_q \propto q^z$ , of a magnetic wave of small wavevector q. For an isotropic and homogeneous metal with  $T_C \rightarrow 0$ , we expect z = 3 at low T (at least for d > 4 - z), a result consistent with inelastic neutron scattering measurements of the spin relaxation spectrum for a number of nearly ferromagnetic cubic d metals (Bernhoeft et al., 1989; Ishikawa et al., 1985).

# 3 Consequences of the long-range interaction in nearly ferromagnetic metals

In the limit  $T_C \rightarrow 0$ , the above model with d = z = 3 leads to the quasiparticle properties described in the introduction which are normally associated with the marginal Fermi liquid state. Thus, the quasiparticles on the Fermi surface are described by an effective mass diverging as  $\ln(T^*/T)$  and a scattering rate proportional to T. This implies a heat capacity of the form  $C \propto T \ln(T^*/T)$  and a resistivity  $\rho \propto T^{5/3}$  to leading order in T. In the resistivity, one factor of T comes from the underlying linear quasiparticle relaxation rate and an additional factor of  $T^{2/3}$  arises from the fact that high q fluctuations are more effective than those at low q in reducing the current. This leads to a weighting factor of  $q_T^2$  in  $\rho$ , where  $q_T$  is a characteristic wavevector satisfying  $T \propto \Gamma_{qT} \propto q_T^z$ . For z = 3, this leads to  $q_T^2 \propto T^{2/3}$  as required. Note that for the corresponding problem of the electron–phonon interaction, we have  $T \propto q_T$  and hence the temperature dependences of  $\rho$  and of the quasiparticle relaxation rate differ by a factor of  $T^2$ instead of  $T^{2/3}$ . The electron-phonon scattering problem differs in other important respects; in particular, the propagation frequency is real rather than imaginary and the phonon spectrum is normally much less strongly temperature dependent than that of magnetic fluctuations.

The above results for C and  $\rho$  hold strictly only in leading order in T and for  $T_C \to 0$ . At elevated temperatures corrections arise from (i) the temperature dependence of  $\Gamma_q \propto q(\kappa^2 + q^2)$ , where  $\kappa(T)$  is the inverse of the magnetic correlation length, and (ii) from the high q form of  $\Gamma_q$  or, effectively, from a cut-off  $\Gamma_{sf}$  in  $\Gamma_q$ .

Numerical analyses based on the standard model for  $\rho(T)$  suggests that the low



Figure 1. The resistivity for MnSi vs temperature at different pressures (5.55 kbar, 8.35 kbar, 10.40 kbar, 11.40 kbar, 12.90 kbar, 13.55 kbar, 14.30 kbar and 15.50 kbar going down starting from the top curve at the far right). The magnetic ordering temperature  $T_C$  (marked by the shoulder in  $\rho$  vs T) decreases towards absolute zero at  $p_c \cong 14.6$  kbar. For  $p > p_c$  a non-Fermi liquid form of  $\rho$  vs T (i.e. a variation  $T^\beta$  with  $\beta \simeq 1.6 < 2$ ) is seen to extend over a wide range (Pfleiderer et al., 1997).

temperature exponent of 5/3 is reached only for T well below (typically two orders of magnitude below) the scale set by the cut off  $T_{sf} = \hbar \Gamma_{sf}/k_B$ . The effective exponent  $\partial \ln \rho / \partial \ln T$  tends to fall monotonically from 5/3 towards 1 due to the effect of the cut-off in  $\Gamma_q$  (see (ii) above) and decreases smoothly towards zero at high  $T/T_{sf}$  due (additionally) to the temperature dependence of  $\Gamma_q$  (see (i) above).

The predictions of this model have been compared with experimental measurements of  $\partial \ln \rho / \partial \ln T$  in the cubic *d* metal MnSi at the critical pressure ( $p_c \approx 15$  kbar) where  $T_C \to 0$  (Pfleiderer et al., 1997). The calculations are based solely on the form of  $\Gamma_q$  inferred from inelastic neutron scattering data at ambient pressures, but with  $\kappa^2(T) \propto 1/\chi(T)$  derived from the temperature dependence of the susceptibility  $\chi(T)$  as measured at  $p_c$ .

The general features of the calculations, including an anomaly at low temperatures which may be traced to the low T peak in  $\chi(T)$ , appear to be in reasonable agreement with the observed behaviour of MnSi (Figs. 1 and 2). We stress, how-



Figure 2. Comparison of measured (points) and calculated (solid lines) logarithmic derivative of the resistivity in MnSi,  $\partial \log \rho / \partial \log T$  plotted vs  $\log_{10}(T(K))$  at the critical pressure. The calculation involves only the measured temperature dependence of the static susceptibility (Pfleiderer et al., 1997), the parameters of the spin fluctuation spectrum inferred from neutron scattering data (Ishikawa et al., 1985) as discussed in the text, and a cut-off wavevector set equal to the Brillouin zone dimension ( $\Gamma X$ ) for the middle line and, respectively, 80% and 120% of  $\Gamma X$  for the lower and upper lines.

ever, that the direct effects of phonon scattering which may be important at high T have been ignored throughout. Also neglected is the phenomenon analogous to phonon drag which may modify the form of  $\rho$  vs T due to spin fluctuations at sufficiently low T. Furthermore, we note that since  $\chi(T)$  in MnSi is not strictly singular at  $p_c$  (Pfleiderer et al., 1997), we expect that the exponent  $\partial \ln \rho / \partial \ln T$  will gradually increase towards 2 with decreasing temperature. The experimental exponent, however, appears to fall somewhat below the predicted value as the temperature is decreased. But within the present experimental accuracy, there is no dramatic or unambiguous discrepancy between the prediction of the above model and the observed form of  $\partial \ln \rho / \partial \ln T$  in MnSi (Pfleiderer et al., 1997) or in the related cubic d metal ZrZn<sub>2</sub> (Grosche et al., 1995).

### 4 Nearly antiferromagnetic f metals

In a search for the limits of applicability of the above standard model for the spinfluctuation mediated quasiparticle interaction, we now turn to the heavy fermion f metals on the border of magnetic transitions at low T. In particular, we consider CePd<sub>2</sub>Si<sub>2</sub> and CeNi<sub>2</sub>Ge<sub>2</sub> which crystallise in a body-centred tetragonal structure that characterises a large family of Ce ternary compounds, including the first of the heavy-fermion superconductors CeCu<sub>2</sub>Si<sub>2</sub> (Steglich et al., 1979). At ambient pressure and below 10 K, CePd<sub>2</sub>Si<sub>2</sub> orders in an antiferromagnetic structure with a weak static moment at low T (Grier et al., 1984).

CeNi<sub>2</sub>Ge<sub>2</sub> has a slightly smaller lattice constant than CePd<sub>2</sub>Si<sub>2</sub> and at ambient pressure exhibits no well-defined magnetic transition. It is reasonable to expect that its behaviour at ambient pressure is similar to that of CePd<sub>2</sub>Si<sub>2</sub> at a pressure somewhat above that required to suppress antiferromagnetic order (Knopp et al., 1988; Fukuhara et al., 1995; Diver, 1996). CeNi<sub>2</sub>Ge<sub>2</sub> therefore provides us with the opportunity to expand the effective range in pressure over which we may explore the behaviour of essentially the same stoichiometric heavy-fermion system close to the boundary of antiferromagnetic order.

As in the case of MnSi, we find that the transition temperature in CePd<sub>2</sub>Si<sub>2</sub> falls continuously towards absolute zero and at the critical pressure ( $p_c \cong 28$  kbar) the temperature dependence of the resistivity is again found to be significantly slower than quadratic (Grosche et al., 1996) (Fig. 3). But in sharp contrast to the case for MnSi, not only  $T_N$ , but also the shoulder of  $\rho$  versus T, shifts rapidly with pressure and in a direction opposite to  $T_N$  (Thompson et al., 1986). At the critical pressure, the shoulder has shifted by nearly an order of magnitude above its position at ambient pressure.

In the wide range opened up between these two characteristic temperatures near  $p_c$ ,  $\rho$  exhibits a remarkable temperature dependence. The resistivity is linear in  $T^{1.2\pm0.1}$  over nearly two decades in temperature down to approximately 0.4 K where our samples with the lowest residual resistivity become superconducting (Fig. 4).

The superconducting regime extends over a relatively narrow pressure range following (and perhaps slightly overlapping with) a regime where  $T_N$  falls towards absolute zero. From the temperature variation of the superconducting upper critical field near  $p_c$ , we infer a low temperature BCS coherence length of approximately 150 Å, a magnitude characteristic of heavy-fermion superconductivity. Related high pressure results have been reported for CeCu<sub>2</sub>Ge<sub>2</sub> (Jaccard et al., 1992) and CeRh<sub>2</sub>Si<sub>2</sub> (Movshovich et al., 1996). What is important in the case of CePd<sub>2</sub>Si<sub>2</sub>, however, is that the normal state above the superconducting transition temperature  $T_s$  does not exhibit a temperature dependent resistivity normally associated



Figure 3. The temperature dependence of the resistivity along the *a* axis of CePd<sub>2</sub>Si<sub>2</sub> at different pressures (Grosche et al., 1996). The Néel temperatures  $T_N$ , marked by arrows, are visible as significant changes in the slope of  $\rho$  vs *T*. The ThCr<sub>2</sub>Si<sub>2</sub> lattice structure of CePd<sub>2</sub>Si and spin configuration below  $T_N$  are illustrated in the insets (Grier et al., 1984). The Ce atoms are on the corners and centre of the tetragonal unit cell, and the Pd atoms are on the cell faces.

with a Fermi liquid state. In some sense, this then represents a form of "high temperature" superconductivity; not high in absolute terms, but in relation to some low temperature scale apparently not yet reached on cooling to  $T_s$ . Among the heavy fermion systems, another extreme but qualitatively different examples of such "high temperature" superconductivity is found in UBe<sub>13</sub> (Ott et al., 1983).

At sufficiently high pressures, we expect to recover a Fermi liquid (quadratic) form of  $\rho$  versus T which is ubiquitous in other paramagnetic heavy fermion metals at low T. As stated earlier, CeNi<sub>2</sub>Ge<sub>2</sub>, with a slightly smaller cell volume than CePd<sub>2</sub>Si<sub>2</sub>, but otherwise with a similar lattice and starting electronic structure, provides us with the opportunity to examine the crossover to the Fermi liquid form of  $\rho$  versus T without the use of very high applied pressures. Initial studies in CeNi<sub>2</sub>Ge<sub>2</sub> suggested a more or less unexceptional behaviour. In particular,  $\rho$  versus T was thought to have a conventional form characteristic of many normal heavy



Figure 4. The temperature-pressure phase diagram of CePd<sub>2</sub>Si<sub>2</sub>. The Néel temperature  $T_N$  falls monotonically towards zero and is nearly linear in pressure before entering a relatively narrow region where superconductivity appears in the millikelvin range (Grosche et al., 1996). The magnetic field dependence of the superconducting transition temperature  $T_s$  exhibits a high slope characteristic of heavy fermion superconductors. Near the critical pressure where  $T_N \rightarrow 0$ , the resistivity is seen to be linear in  $T^{1.2\pm0.1}$  over nearly two decades in temperature (inset) (Grosche et al., 1996).

fermion systems. But more detailed studies in samples with low residual resistivities have revealed that  $\rho$  does not exhibit a simple quadratic temperature variation except perhaps below one or two hundred mK and in fact varies as  $\rho \sim T^{1.4\pm0.1}$ over a decade below several degrees K (Steglich, 1996; Diver, 1996). This anomalous behaviour of the temperature dependence of the resistivity appears to extend up to 10 kbar and beyond where a new superconducting instability is observed (Grosche, 1997; Carter et al., unpublished). Also, we note that in both compounds the value of the anomalous exponent of  $\rho$  versus T appears to be quite sensitive in particular to sample perfection (Carter et al., unpublished).

The nature of the anisotropic spin fluctuation spectra of  $CePd_2Si_2$  and  $CeNi_2Ge_2$ are not yet sufficiently well known to enable us to carry out a quantitative analysis analogous to that presented for the *d*-metal ferromagnets in the previous section. Further, the validity of the naive extension of this model to antiferromagnetic systems (for which z is taken to be 2 instead of 3) is not clearly self evident (Hlubina and Rice, 1995). For the very simplest case with d = 3 and z = 2, the exponent  $\partial \ln \rho / \partial \ln T$  falls monotonically from 3/2 for  $T \gg T_{sf}$  towards zero for  $T \ll T_{sf}$ . The observation of a low temperature exponent of less than 3/2 in CePd<sub>2</sub>Si<sub>2</sub> at p<sub>c</sub> is not necessarily inconsistent with this model since convergence of  $\partial \ln \rho / \partial \ln T$  to 3/2 is found to be very slow. But the locking in of the experimental exponent to a fixed value over a wide temperature range is not a feature of the present spin fluctuation model. Also difficult to understand within this same framework is the non-Fermi liquid form of  $\rho$  versus T at still higher pressures above  $p_c$  in CePd<sub>2</sub>Si<sub>2</sub>, or in the smaller volume relative CeNi<sub>2</sub>Ge<sub>2</sub> at ambient or low pressures and above one or two hundred mK.

It is not yet clear whether a consistent description of the above findings can be given in terms of a refined version of the model developed for the d-metal ferromagnets or whether a radically different approach is required. In the f heavy fermion systems, in contrast to typical d metals, there may be an ambiguity in the spin-fluctuation theory as it is conventionally formulated. It is perhaps unclear in our systems whether the Fermi surface close to which the relevant quasiparticles are excited is that formed by the "conduction electrons" together with the f electrons, as suggested by de Haas–van Alphen studies on a number of normal heavy fermion compounds, or by the conduction electrons alone as is often assumed in "intermediate temperature" descriptions. What is more, the usual assumption that the strength of the coupling of the quasiparticles to the exchange field is weakly temperature dependent may in these highly correlated systems seriously break down in the temperature range of interest.

### 5 Conclusions

The idealised model for describing nearly ferromagnetic d metals, such as MnSi and ZrZn<sub>2</sub>, near the critical point  $T_C \rightarrow 0$ , appears to be that of a marginal Fermi liquid which has also been invoked in theoretical treatments of the coupling of electrons to transverse photons and in the study of nuclear matter (Baym and Pethick, 1991). In both cases, the starting picture remains that of fermion quasiparticles excited above a normal Fermi surface. In more extreme cases, an altogether different starting point may be required.

It is conceivable that this is the case in some of the more strongly correlated electron systems among the heavy fermion compounds (see also Morin et al., 1988; Löhneysen et al., 1994; Seaman et al., 1991; Andraka and Stewart, 1993; Tsvelik and Reizer, 1993; Coleman, 1995b). In particular, we have noted that a naive extension of the spin fluctuation model used for the *d*-metal ferromagnets cannot readily account for the curious locking into a fixed exponent  $\partial \ln \rho / \partial \ln T$  over nearly two decades in temperature in CePd<sub>2</sub>Si<sub>2</sub> near the critical pressure nor in CeNi<sub>2</sub>Ge<sub>2</sub> at ambient pressure.

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