INTERNATIONAL JOURNAL OF ENERGY RESEARCH
Int. J. Energy Res., **22**, 805–812 (1998)

EFFICIENCY BOUND OF A SOLAR-DRIVEN STIRLING HEAT ENGINE SYSTEM

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SUMMARY

A solar-driven Stirling engine is modelled as a combined system which consists of a solar collector and a Stirling engine. The performance of the system is investigated, based on the linearized heat loss model of the solar collector and the irreverisible cycle model of the Stirling engine affected by finite-rate heat transfer and regenerative losses. The maximum efficiency of the system and the optimal operating temperature of the solar collector are determined. Moreover, it is pointed out that the investigation method in the present paper is valid for other heat loss models of the solar collector as well, and the results obtained are also valid for a solar-driven Ericsson engine system using an ideal gas as its engine work substance. © 1998 John Wiley & Sons, Ltd.

KEY WORDS efficiency bound; Stirling engine; solar collector; regenerative loss; heat transfer loss; finite-time thermodynamics; optimization

1. INTRODUCTION

The Stirling cycle is one of the important standard air cycles for heat engines (Zemansky, 1968; Holman, 1980). Its main advantages are that the cycle may be driven by a wide variety of fuels, and it offers the opportunity for high efficiency (Holman, 1980; Ayres and Mckenna, 1972; Wu, 1994). Since the Philips company revived the Stirling engine and produced functional engines (Zemansky, 1968; Witteveen, 1966), Stirling engines have once again become the subject of great interest and research, and have been seriously considered for a variety of uses. For example, Stirling engines driven by solar energy may be used in space-power applications (Noyes, 1990; Shaltens, 1991; Badescu, 1992).

In recent years finite-time thermodynamics has been used successfully to study the optimal performance of solar-driven energy systems (Yan, 1992; Chen, 1992; Yan and Chen, 1994). The results obtained have more realistic instructive significance for the optimal design of real solar-driven systems than those derived from traditional equilibrium thermodynamics. In the present paper we will determine the efficiency bound of a solar-driven heat engine system with the irreversibilities of heat conduction and regenerative losses by means of finite-time thermodynamics. In addition we derive the optimal temperature of the solar collector.

2. EFFICIENCY OF A STIRLING ENGINE

The Stirling cycle consists of two isothermal branches and two constant generalized coordinate branches such as constant-volume or isomagnetic branches (Chen and Yan, 1993). When the working substance in the

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CCC 0363–907X/98/090805–8$17.50

Received 5 September 1997
Accepted 22 December 1997

Stirling cycle is assumed to be an ideal gas, the amounts of heat Q_1 and Q_2 absorbed from the heat reservoir at temperature T_h and released to the heat sink at temperature T_c by the working substance during the two isothermal branches are

$$Q_1 = nRT_1 \ln(V_2/V_1) \tag{1}$$

and

$$Q_2 = nRT_2 \ln(V_2/V_1) \tag{2}$$

respectively, where n is the mole number of the working substance, R is the universal gas constant, T_1 and T_2 are the temperatures of the working substance during the high- and low-temperature isothermal branches, and V_1 and V_2 are the volumes of the working substance along the constant-volume heating and cooling branches, as shown in Figure 1.

Invariably, there are thermal resistances between the working substance and the external heat reservoirs in the Stirling engine. In order to obtain a certain power output, the temperatures of the working substance must therefore be different from those of the heat reservoirs. When heat transfer obeys a linear law (Andresen, 1983), Q_1 and Q_2 may be written as

$$Q_1 = k_1(T_h - T_1)t_1 \tag{3}$$

and

$$Q_2 = k_2(T_2 - T_c)t_2 \tag{4}$$

respectively, where k_1 and k_2 are the thermal conductances between the working substance and the heat reservoirs at temperatures T_h and T_c, and t_1 and t_2 are the times spent on the two isothermal branches at temperatures T_1 and T_2, respectively.

It should be pointed out that also the regenerative branches are affected by internal thermal resistances to and from the thermal regenerator. Thus, regenerative losses are inevitable. One may quantify these regenerative losses by (Howell and Bannerot, 1977)

$$\Delta Q = xnC(T_1 - T_2) \tag{5}$$

where C is the heat capacity of the working substance per mole partaking in the regenerative branches, and x is the fractional deviation from ideal regeneration. When $x = 0$ the Stirling cycle operators with ideal (complete) regeneration.

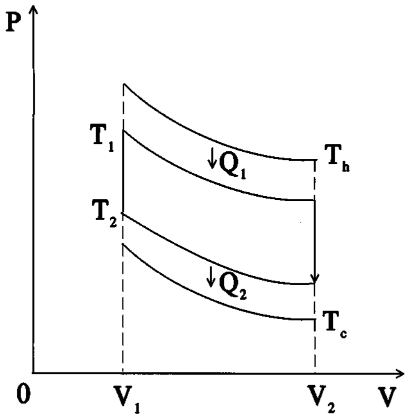

Figure 1. PV-diagram of a Stirling engine

Owing to the existence of regenerative losses, the net amounts of heat Q_h and Q_c absorbed from the heat source at temperature T_h and released to the heat sink at temperature T_c by the working substance per cycle are given by

$$Q_h = Q_1 + \Delta Q \tag{6}$$

and

$$Q_c = Q_2 + \Delta Q \tag{7}$$

respectively.

For the convenience of analysis it is assumed that the time spent on the regenerative branches is proportional to that of the isothermal branches, i.e.

$$t_{re} = b(t_1 + t_2) \tag{8}$$

where b is a constant. Thus, the cyclic period is given by

$$t = t_1 + t_2 + t_{re} = (1 + b)(t_1 + t_2) \tag{9}$$

From equations (1)–(7) and (9) we can find the efficiency of the Stirling engine

$$\eta_h = \frac{T_1 - T_2}{T_1 + a(T_1 - T_2)} \tag{10}$$

and the average rate of heat supply

$$q_h = \frac{Q_h}{t} = \frac{T_1 + a(T_1 - T_2)}{(1 + b)[T_1/(k_1(T_h - T_1)) + T_2/(k_2(T_2 - T_c))]} \tag{11}$$

where $a = xC/[R \ln(V_2/V_1)]$. It can be proven from equations (10) and (11) that for a given q_h, the maximum efficiency of the Stirling engine is given by

$$\eta_h = \frac{q_h/K + (a-1)T_h - aT_c + T_e}{2aT_h + a[q_h/K + (a-1)T_h - aT_c + T_e]} \tag{12}$$

where

$$T_e = \sqrt{[(1+a)T_h + aT_c - q_h/K]^2 - 4(1+a)aT_hT_c} \tag{13}$$

and

$$K = \frac{k_1}{(1 + b)(1 + \sqrt{k_1/k_2})^2} \tag{14}$$

When $K \to \infty$ (no resistance to the reservoirs), equation (12) may be simplified to

$$\eta_h = \frac{T_h - T_c}{T_h + a(T_h - T_c)} \tag{15}$$

Equation (15) is just the result of Howell and Bannerot (1977) which considers only regenerative losses. This shows that by using equation (12) to study the performance of a solar-driven Stirling engine system, one can obtain more realistic conclusions because not only regenerative losses are considered but also the irreversibility of finite-rate heat transfer in/out of the Stirling engine.

3. SYSTEM EFFICIENCY

Figure 2 presents a simplified view of a solar-driven Stirling heat engine system which consists of a solar collector and a Stirling engine. Here q_h is the useful thermal power provided by the solar collector, q_c is the rate of heat transfer from the Stirling engine to the heat sink at temperature T_c, and P is the power output of the Stirling engine.

According to the definition of the efficiency of a solar-driven Stirling engine system, we have

$$\eta = \frac{P}{I_s A_c} = \frac{q_h}{I_s A_c}\frac{P}{q_h} = \eta_s \eta_h \tag{16}$$

where $\eta_s = q_h/I_s A_c$ is the thermal efficiency of the solar collector, I_s is the direct solar flux, and A_c is the collector projected area.

For a solar collector that provides the useful thermal power q_h at temperature T_h, the energy balance may be written as (Howell and Bannerot, 1977; Bejan, 1988)

$$q_h = \tau\alpha_s I_s A_c - \varepsilon_b \sigma T_h^4 A_b - U_b A_b (T_h - T_c) \tag{17}$$

where τ is the transmittance of the cover-plate assembly, α_s is the effective solar absorptance of the base of the collector, ε_b is the effective infrared emittance of the absorber plate, U_b is the overall convective heat transfer coefficient referenced to the base-plate area A_b, and T_h is the operating temperature of the collector. At low and intermediate temperatures heat losses are dominated by convection and/or conduction and U_b may be replaced by another constant U_B so as to include the radiation losses in the linearized heat loss term (Kandpal *et al.*, 1983). In such a case the efficiency of the solar collector may be given by (Yan and Chen, 1994; Bejan *et al.*, 1981; Bejan, 1982; Gordon, 1988)

$$\eta_s = \frac{q_h}{I_s A_c} = \tau\alpha_s - \frac{U_B A_b}{I_s A_c}(T_h - T_c) = \frac{U}{I_s}(T_s - T_h) \tag{18}$$

where $U = U_B A_b / A_c$ and

$$T_s = \tau\alpha_s I_s A_c / U_B A_b + T_c \tag{19}$$

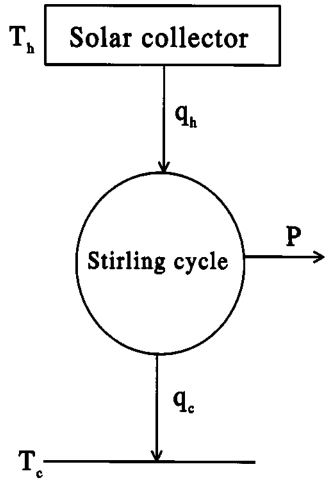

Figure 2. A solar-driven Stirling engine system

is the collector temperature under no-flow conditions ($q_h = 0$) and referred to as the collector stagnation temperature (Gordon, 1988).

Using equations (18) and (12) we obtain the efficiency of the complete solar-driven Stirling engine system

$$\eta = \frac{U}{I_s}(T_s - T_h)\frac{a_1(T_s - T_h) + (a-1)T_h - aT_c + T^*}{2aT_h + a[a_1(T_s - T_h) + (a-1)T_h - aT_c + T^*]} \tag{20}$$

where $a_1 = UA_c/K$ and

$$T^* = \sqrt{[(1+a)T_h + aT_c - a_1(T_s - T_h)]^2 - 4(1+a)aT_hT_c} \tag{21}$$

4. MAXIMUM EFFICIENCY AND OPTIMAL VALUE OF T_h

It is seen from equation (20) that when $T_h = (T_c + a_1T_s)/(1 + a_1)$ or $T_h = T_s$, then $\eta = 0$ and no power is produced. This implies that when T_h is equal to some value between these two roots, η must have a maximum, as shown in Figure 3. Obviously, the optimal value of the operating temperature of the solar collector is at the maximum efficiency of the solar-driven Stirling engine system, determined by the following equation:

$$2AT_h^2 - 2T_s(T' + T^*) - (T' - AT_h + T^*)^2 + 2(T_s - T_h)T_hB = 0 \tag{22}$$

where $A = 1 - a + a_1$, $T' = a_1T_s - aT_c$, and $B = \{[(1 + a + a_1)T_h - T'](1 + a + a_1) - 2(1 + a)aT_c\}/T^*$.

Starting from equation (22) we can further obtain the curve of the optimal operating temperature of the solar collector versus the relative parameters of the system, as shown in Figure 4. It is apparent that the optimal operating temperature $T_{h,opt}$ of the solar collector decreases as the regenerator inefficiency parameter a increases for a given a_1, while $T_{h,opt}$ increases as the thermal transfer inefficiency parameter a_1 increases for a given a. Clearly, the larger the parameter a is, the larger the regenerative losses are. In order to reduce such losses, the operating temperature T_h of the solar collector should be decreased. The larger the parameter a_1 is, the larger the thermal resistances between the engine working substance and the two heat

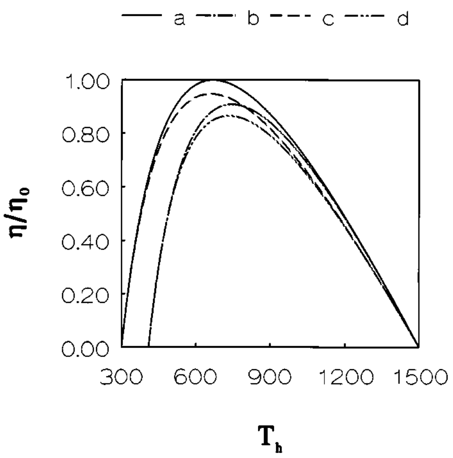

Figure 3. The efficiency η (normalized to its maximum) of the system versus the operating temperature T_h of the solar collector. Plots are presented for $T_s = 1500$ K, $T_c = 300$ K, and $\eta_0 = (U/I_s)(\sqrt{T_s} - \sqrt{T_c})^2$. The curves correspond to (a) $a = a_1 = 0$; (b) $a = 0$ and $a_1 = 0{\cdot}1$; (c) $a = 0{\cdot}1$ and $a_1 = 0$; and (d) $a = a_1 = 0{\cdot}1$

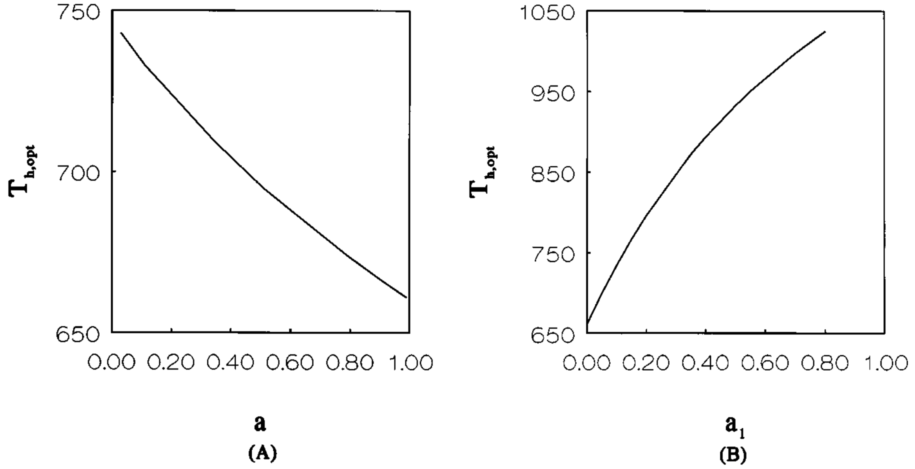

Figure 4. The optimal operating temperature $T_{h,opt}$ vs. the parameters a(A) and a_1(B). $a_1 = 0{\cdot}1$ and $a = 0{\cdot}1$ are adotped in (A) and (B), respectively. Plots are presented for $T_s = 1500$ K and $T_c = 300$ K

reservoirs are and the lower the efficiency of the Stirling engine is. In order to increase the efficiency of the engine, the operating temperature of the solar collector should be increased.

When $K \to \infty$, $a_1 = 0$. Equations (20) and (22) may then be simplified to

$$\eta = \frac{U}{I_s}(T_s - T_h)\frac{T_h - T_c}{T_h + a(T_h - T_c)} \tag{23}$$

and

$$T_h = \frac{aT_c + \sqrt{T_sT_c + aT_c(T_s - T_c)}}{1 + a} \tag{24}$$

Substituting T_h into equation (23) we obtain the maximum efficiency

$$\eta_{max} = \frac{U}{I_s}\frac{\sqrt{T_sT_c + aT_c(T_s - T_c)} - T_c}{(1 + a)^2}\left(\frac{(1 + a)T_s - aT_c}{\sqrt{T_sT_c + aT_c(T_s - T_c)}} - 1\right) \tag{25}$$

Equations (23)–(25) are just the results derived for the Stirling engine coupled reversibly to its reservoirs (Howell and Bannerot, 1977).

When $x = 0$, $a = 0$, equations (12) and (20) may be simplified to

$$\eta_h = 1 - \frac{T_c}{T_h - q_h/K} \tag{26}$$

and

$$\eta = \frac{U}{I_s}(T_s - T_h)\left(1 - \frac{T_c}{T_h - a_1(T_s - T_h)}\right) \tag{27}$$

respectively. Using equation (27) we obtain the maximum efficiency

$$\eta_{max} = \frac{U}{I_s}\frac{(\sqrt{T_s} - \sqrt{T_c})^2}{1 + a_1} \tag{28}$$

with the corresponding operating temperature

$$T_h = \frac{\sqrt{T_s T_c} + a_1 T_s}{1 + a_1} \tag{29}$$

It is worthwhile to notice that equations (26)–(29) are valid for any solar-driven reversible heat engine system as long as the time spent on the two temperature-changing branches in the cycle is proportional to that of the two isothermal branches. This shows clearly that if the regenerative losses were not considered in the investigation of the Stirling engine systems, we would not have obtained new conclusions which are intrinsically different from those of the reversible engine systems.

5. CONCLUSIONS

Regeneration is one of the important characteristics of the Stirling cycle. The importance of this paper lies in a new cycle model which can describe the general characteristics of a solar-driven Stirling engine system with the irreversibilities of regenerative losses and heat conduction. The maximum efficiency of the system and the optimal operating temperature of the solar collector are determined.

The above results can be generalized to be suitable for any two-heat-source cycle system driven by solar energy so long as regenerative losses may be described by equation (5) and V_2/V_1 is replaced by the appropriate ratio of changing coordinates. For example, the above results will be suitable for a solar-driven Ericsson engine system using an ideal gas as its engine working substances so long as V_2/V_1 is replaced by P_2/P_1, where P_1 and P_2 are the pressures on the two constant-pressure branches in the Ericsson cycle. As another example, the above results are also suitable for a solar-driven Stirling engine system using a Van der Waals gas as its engine working substance so long as V_2/V_1 is replaced by $(V_2 - nb_1)/(V_1 - nb_1)$, where b_1 is a constant in the equation of state of Van der Waals gases (Gutkowicz-Krusin *et al.*, 1978).

When the radiation heat losses in the solar collector are not negligible, equations (17) and (12) must be used directly to study the optimal performance of a solar-driven Stirling engine system by means of the same method of the present paper. In the limit of $a_1 \to 0$ the results for an ideally coupled solar-driven Stirling engine in Howell and Bannerot (1977) can be recovered. Finally, it is worth while to point out that the above method may be used in further investigations of other solar-driven cycle systems.

ACKNOWLEDGEMENTS

This work was supported by both the National Natural Science Foundation of China and the EU Copernicus Program under Grant No. CIPA-CT92-4026.

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