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OPTIMAL ANALYSIS OF PRIMARY PERFORMANCE PARAMETERS FOR AN ENDOREVERSIBLE ABSORPTION HEAT PUMP

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Abstract—The cycle model of a heat-engine-driven heat pump is used to study the performance of an absorption heat pump affected by heat resistances. The coefficient of performance of the absorption heat pump is adopted to be the objective function for optimization. The optimal regions of the coefficient of performance and the specific heating load are determined. The optimal relations between the heat transfer areas of the four heat exchangers involved and the coefficient of performance, or the specific heating load of an absorption heat pump, are obtained. Problems concerning the optimal design of an absorption heat pump are also discussed.

1. INTRODUCTION

Many industrial processes reject heat to the surroundings at a temperature high enough above ambient temperature and in sufficient quantities to make heat recovery economically attractive. Considering that the most efficient modern steam power plants are operating at only 40% efficiency, the amount of waste heat rejected to the ambient could easily amount to 69% or more of the total energy consumed in many industrial processes. Conversely, recovered waste heat can directly reduce the energy cost per unit of industrial product and thermal pollution of the atmosphere and water can also be produced as a by-product of waste heat recovery; this has resulted in the advent of various novel waste heat recovery devices.

Among the different systems currently adopted for the recovery of industrial waste heat, absorption heat pumps are considered to be the most competitive [1]. In recent years the practical exploitation of absorption heat pumps has appeared in Western Europe, Japan and America, as well as other countries; this has encouraged the theoretical investigation of absorption heat pumps [2–6].

The main purpose of this paper is to use the theory of finite time thermodynamics to analyze the effect of thermal resistances on the performance of an absorption heat pump system. This includes optimization of the primary performance parameters of the system, such as the coefficient of performance, specific heating load and heat transfer areas of heat exchangers.

2. AN ABSORPTION HEAT PUMP SYSTEM

The primary components of an absorption heat pump are a generator, an absorber, a condenser and an evaporator [3, 4, 7], as shown in Fig. 1. Here q_h and q_l are, respectively, the rates of heat transfer from the heat source at a high temperature , T_h, to the generator and from the heat sink (ambient) at a low temperature, T_l, to the evaporator; while q_a and q_c are, respectively, the rates of heat transfer from the absorber and the condenser to the heated space at an intermediate temperature, T_p. The heating load, q_p, of an absorption heat pump is thus the sum of these: $q_p = q_a + q_c$. Work required for mechanical pumping within the system is negligible relative to the

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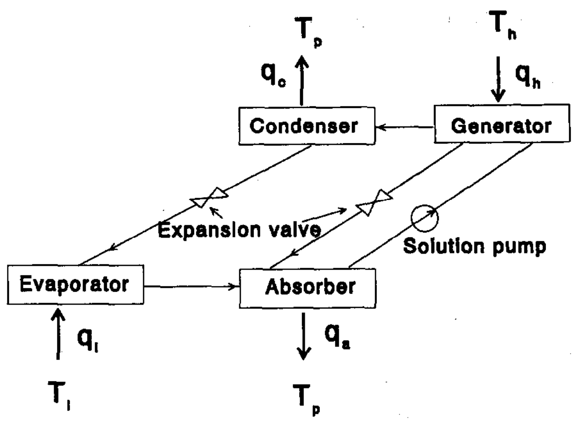

Fig. 1. An absorption heat pump system.

energy input to the generator and therefore is often neglected for the purpose of analysis [2, 3]. According to the first law of thermodynamics one has

$$q_h + q_l = q_a + q_c. \tag{1}$$

In practical absorption heat pumps there exist many sources of irreversibility, such as finite-rate heat transfer, heat leakage, friction, dissipation in the working fluid and so on. In order to obtain some significant analytic solutions of primary performance parameters, we only consider the effect of heat resistances on the performance of the absorption heat pump. In such a case an absorption heat pump may be treated as a combined cycle in which an endoreversible heat engine operating between the heat reservoirs at temperatures T_h and T_p drives an endoreversible heat pump operating between the heat reservoirs at temperatures T_p and T_l, as shown in Fig. 2, where T_1, T_2, T_3 and T_4 are, respectively, the temperatures of the working fluid in the generator, absorber, condenser, and evaporator and p is the power output of the heat engine.

According to the usual definition of the coefficient of performance ψ of an absorption heat pump [1, 2, 5] we have

$$\psi = \frac{q_p}{q_h} = \frac{q_a + q_c}{q_h} = \frac{q_h - p}{q_h} + \frac{p}{q_h}\frac{q_c}{p} = 1 - \eta + \eta\phi, \tag{2}$$

where $\eta = p/q_h$ is the efficiency of the heat engine operating between the heat reservoirs at temperatures T_h and T_p and $\phi = q_c/p$ is the coefficient of performance of the heat pump operating between the heat reservoirs at temperatures T_p and T_l.

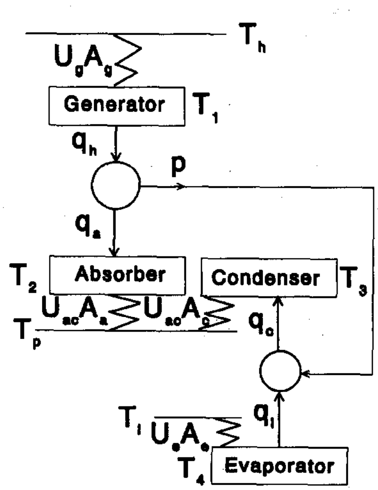

Fig. 2. The equivalent cycle model of an endoreversible absorption heat pump.

3. THE OPTIMAL COEFFICIENT OF PERFORMANCE

When heat transfer obeys a linear law [8–12], one has

$$q_h = U_g A_g (T_h - T_1), \tag{3}$$

$$q_a = U_{ac} A_a (T_2 - T_p), \tag{4}$$

$$q_c = U_{av} A_c (T_3 - T_p), \tag{4}$$

$$q_l = U_e A_e (T_l - T_4), \tag{6}$$

where A_g, A_a, A_c and A_e are, respectively, the heat transfer areas of the generator, absorber, condenser and evaporator; U_g and U_e are, respectively, the heat transfer coefficients of the generator and evaporator and U_{ac} is the common heat transfer coefficient of the absorber and condenser. The latter is a reasonable assumption because the working fluid in the absorber and the condenser exchanges heat with the same heat reservoir at temperature T_p.

The total heat transfer areas of the heat engine and heat pumps are thus, respectively,

$$A_h = A_g + A_a \tag{7}$$

and

$$A_p = A_c + A_e, \tag{8}$$

making the total heat transfer area of the system

$$A = A_h + A_p = A_g + A_a + A_c + A_e. \tag{9}$$

We can prove from equations (3), (4) and (7) that the optimal distribution of heat transfer areas A_g and A_a is [13, 14]

$$A_g / A_a = \sqrt{U_{ac}/U_g}. \tag{10}$$

Then the efficiency of the heat engine operating between the heat reservoirs at temperature T_h and T_p is given by [13, 14]

$$\eta = 1 - T_p / [T_h - q_h/(U_h A_h)] \tag{11}$$

for a specified heat input rate q_h and a given total heat transfer area A_h, where $U_h = U_g U_{ac}/(\sqrt{U_g} + \sqrt{U_{ac}})^2$.

Similarly, we can also prove from equations (5), (6) and (8) that when the heat transfer areas A_c and A_e are chosen such that

$$A_c / A_e = \sqrt{U_e / U_{ac}}, \tag{12}$$

then the coefficient of performance of the heat pump operating between the heat reservoirs at temperatures T_p and T_l is optimal and given by

$$\phi = \frac{T_p + q_c/(U_p A_p)}{T_p - T_l + q_c/(U_p A_p)} \tag{13}$$

for a specified q_c and a given total heat transfer area A_p, where $U_p = U_{ac} U_e / (\sqrt{U_{ac}} + \sqrt{U_e})^2$.

Substituting equations (11) and (13) into equation (2) and using equation (9) and the following energy conservation relation:

$$q_l = q_c(1 - 1/\phi) = q_p(1 - 1/\psi), \tag{14}$$

we obtain an expression for the coefficient of performance for an endoreversible absorption heat pump:

$$\psi = \frac{T_p}{T_h - \dfrac{q_p}{U_h \psi (A - A_p)}} + \left[1 - \frac{T_p}{T_h - \dfrac{q_p}{U_h \psi (A - A_p)}}\right] \frac{T_p}{T_p - T_l + q_p \dfrac{1 - 1/\psi}{U_p A_p}}. \tag{15}$$

Starting from equation (15), we can prove that for a specified heating load, q_p, of an absorption heat pump and a given total heat transfer area, A, of the four heat exchangers in the system, the coefficient of performance of the entire system attains its maximum when the combined heat transfer area of the heat pump, A_p, is determined by

$$A_p = A\frac{T_h - T_p - [1 + C(\psi - 1)]\Pi/(U_h\psi)}{T_h - T_p + (T_p - T_l)/C}, \tag{16}$$

where $C = \sqrt{(U_h/U_p)}$, and $\Pi = q_p/A$ is the specific heating load [15] of an absorption heat pump. It represents the heating load per unit total heat transfer area.

Substituting equation (16) into equation (15), we obtain the optimal coefficient of performance of an endoreversible absorption heat pump:

$$\psi = \frac{U_h T_p(T_h - T_l)\psi + (C-1)[1 + C(\psi - 1)]T_p\Pi}{U_h T_h(T_p - T_l)\psi + [T_l + (C-1)T_p + C^2(\psi - 1)T_h]\Pi} \tag{17}$$

for a given specific heating load Π. Equation (17) is the same as the general optimum relation [16] derived from the cycle model of an endoreversible three-heat-source heat pump. So far we have proven that the optimal performance of an endoreversible absorption heat pump may be analyzed by an equivalent combined cycle, shown in Fig. 2. However, it is more important that, by using an equivalent combined cycle to analyze the performance of an endoreversible absorption heat pump, we can determine more exactly the optimal regions of the coefficient of performance and specific heating load and give some optimum rules of other primary performance parameters.

4. OPTIMAL REGIONS OF ψ AND Π

Equation (17) shows that the optimal coefficient of performance of an endoreversible absorption heat pump is a monotonically decreasing function of the specific heating load, as shown in Fig. 3. When an absorption heat pump attains its reversible coefficient of performance [1, 17],

$$\psi_r = \frac{T_h - T_l}{T_h}\frac{T_p}{T_p - T_l}, \tag{18}$$

its specific heating loss is equal to zero. This indicates that the reversible coefficient of performance of an absorption heat pump is of very limited practical value, because real absorption heat pumps are always required to have a certain heating load.

When its specific heating load attains its maximum,

$$\Pi_{max} = U_h(T_h - T_p), \tag{19}$$

the coefficient of performance is equal to 1, i.e. the heat delivered to the heated space equals the heat drawn from the source. Because $T_h > T_p$, one can instead use the heat source at temperature

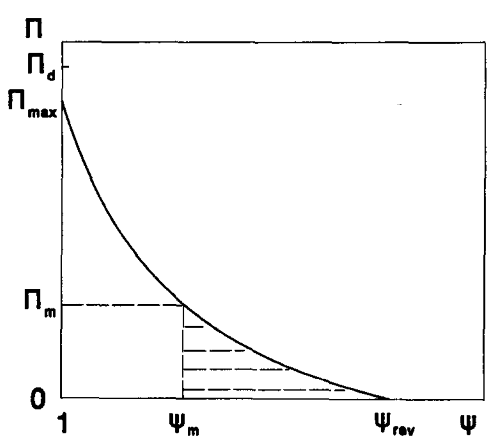

Fig. 3. The specific heating load Π vs the coefficient of performance ψ for an endoreversible absorption heat pump. Direct head conduction from the heat source to the heated space is marked as Π_d. The more efficient branch of operation of the driving heat engine is shaded.

T_h to supply heat directly to the heated space at temperature T_p. It is easily accomplished to make the heat transfer coefficient and the area between the heat source and the heated space, respectively, be equal to U_g and A. Then the rate of heat transfer per unit heat transfer area Π_d in the process of direct conduction of heat is

$$\Pi_d = U_g(T_h - T_p) > \Pi_{max}, \tag{20}$$

as shown in Fig. 3. This shows clearly that absorption heat pumps operating at maximum specific heating load are not as effective as direct conduction of heat, so absorption heat pumps should not be operated in the neighbourhood of $\psi = 1$. Hence, both the coefficient of performance and the specific heating load of an absorption heat pump must be considered together.

From equations (11), (2), (9), (16) and (17) we obtain the relation between the efficiency of the heat engine and the coefficient of performance of the system:

$$\eta = \frac{(\psi - 1)(1 + C_1)}{(\psi - 1)C_2 - 1 + CT_p/(T_p - T_1) - (C - 1)\psi T_h/(T_h - T_1)}, \tag{21}$$

where $C_1 = (T_h - T_p)[C^2/(T_p - T_1) - (C - 1)^2/(T_h - T_1)]$ and $C_2 = T_h[C^2/(T_p - T_1) - C - 1)^2/(T_h - T_1)]$ are constants. It is well known that for an endoreversible heat engine operating between two heat reservoirs at temperatures T_h and T_p, its efficiency at maximum power output [18, 19], p_{max}, or maximum specific power output [20] is given by

$$\eta_{CA} = 1 - \sqrt{T_p/T_h}. \tag{22}$$

When $p < p_{max}$, the efficiency η of the endoreversible heat engine may attain two different values for a given p, one being smaller than η_{CA} and the other larger than η_{CA}, as shown in Fig. 4. Only if the efficiency of the endoreversible heat engine is situated in the shaded region between the efficiency η_{CA} and the efficiency η_{rev} of a reversible heat engine,

$$\eta_{rev} > \eta \geqslant \eta_{CA}, \tag{23}$$

can the endoreversible heat engine be operated optimally.

The condition that absorption heat pumps are required to operate in the optimal region of the coefficient of performance ψ required the efficiency of the endoreversible heat engine in the system satisfying equation (23). Thus solving equations (21) and (23), we find that the optimal region of the coefficient of performance for an endoreversible absorption heat pump is

$$\psi \geqslant \frac{1 + C_1 + \eta_{CA}[CT_p/(T_p - T_1) - 1 - C_2]}{1 + C_1 + \eta_{CA}[(C - 1)T_h/(T_h - T_1) - C_2]} = \psi_m. \tag{24}$$

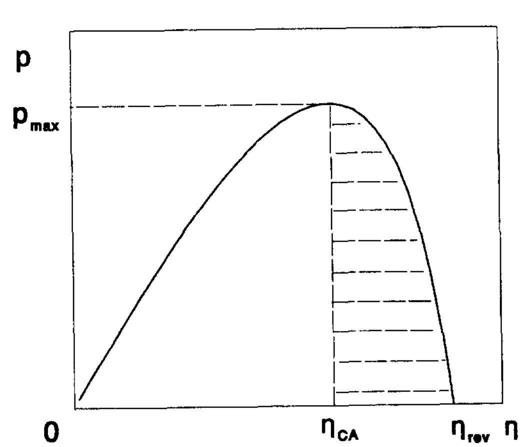

Fig. 4. The power output p vs the efficiency η for an endoreversible heat engine. The shaded region is the more efficient branch of operation for a given power production.

Substituting equation (24) into equation (17), we can determine the corresponding optimal region of the specific heating load:

$$\Pi \leqslant U_h \psi_m \frac{T_p(T_h - T_l) - \psi_m T_h (T_p - T_l)}{\psi_m T_l + (C-1)^2(1-\psi_m)T_p + C^2 \psi_m(\psi_m - 1)T_h} = \Pi_m. \tag{25}$$

This area is shown shaded in Fig. 3.

Equations (24) and (25) are two important conclusions for an endoreversible absorption heat pump, with ψ_m and Π_m being very useful parameters. Obviously, if the coefficient of performance $\psi < \psi_m$ and the specific heating load $\Pi > \Pi_m$, the efficiency of the endoreversible heat engine in the system is smaller than η_{CA}. As mentioned above, the endoreversible heat engine with an efficiency $\eta < \eta_{CA}$ is not situated in the optimal working region so that the whole system cannot be operated optimally, i.e. most efficiently. It is thus clear that ψ_m determines a lower bound for the optimal coefficient of performance and Π_m determines an upper bound for the specific heating load. In the symmetric case, when the heat transfer coefficients $U_h = U_p$, ψ_m and Π_m can be simply written as

$$\psi_m = \frac{T_h - T_l}{T_h} \frac{T_p}{T_p - T_l\sqrt{T_p/T_h}} \tag{26}$$

and

$$\Pi_m = U_h(\sqrt{T_h T_p} - T_p). \tag{27}$$

Comparing equations (26) and (27) with equations (18) and (19), we obtain

$$\psi_m = \psi_r \frac{1 - T_l/T_p}{1 - T_l/\sqrt{T_h T_p}} \tag{28}$$

and

$$\Pi_m = \frac{\Pi_{max}}{1 + \sqrt{T_h/T_p}} < \frac{\Pi_{max}}{2}, \tag{29}$$

respectively. From equation (29) we can easily generate the curve of Π_m/Π_{max} vs T_h/T_p, as shown in Fig. 5. This shows clearly that the rational specific heating load of an endoreversible absorption heat pump should be smaller than half of the maximum specific heating load, because then the efficiency of the endoreversible heat engine inside the system will be greater than η_{CA}.

The above results are significant for the optimal design and operation of real absorption heat pumps.

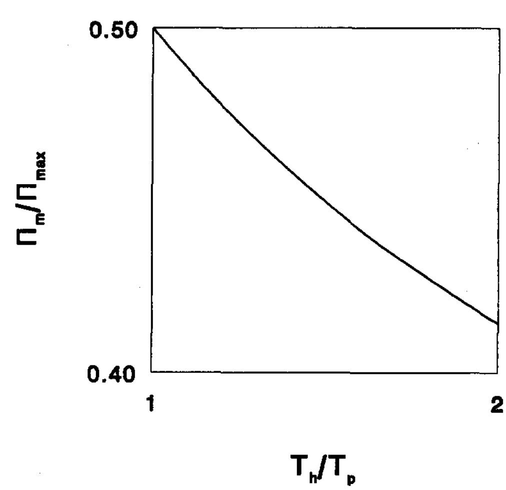

Fig. 5. The ratio of the specific heating loads Π_m/Π_{max} vs the ratio of the reservoir temperatures T_h/T_p for an endoreversible absorption heat pump with $U_h = U_p$.

5. OPTIMAL DISTRIBUTION OF HEAT EXCHANGER AREAS

Solving equations (7)–(10), (12) and (16) we find the optimal heat transfer areas:

$$\frac{A_g}{A}=\sqrt{\frac{U_h}{U_g}\frac{(T_p-T_l)/C+[1+C(\psi-1)]\Pi/(U_h\psi)}{T_h-T_p+(T_p-T_l)/C}}, \tag{30}$$

$$\frac{A_a}{A}=\sqrt{\frac{U_h}{U_{ac}}\frac{(T_p-T_l)/C+[1+C(\psi-1)]\Pi/(U_h\psi)}{T_h-T_p+(T_p-T_l)/C}}, \tag{31}$$

$$\frac{A_c}{A}=\sqrt{\frac{U_p}{U_{ac}}\frac{T_h-T_p-[1+C(\psi-1)]\Pi/(U_h\psi)}{T_h-T_p+(T_p-T_l)/C}}, \tag{32}$$

$$\frac{A_e}{A}=\sqrt{\frac{U_p}{U_e}\frac{T_h-T_p-[1+C(\psi-1)]\Pi/(U_h\psi)}{T_h-T_p+(T_p-T_l)/C}}. \tag{33}$$

Equations (30)–(33) result in a very simple relation for the optimal distribution of heat transfer areas of the four heat exchangers:

$$\sqrt{U_g}A_g+\sqrt{U_e}A_e=\sqrt{U_{ac}}(A_a+A_c). \tag{34}$$

Solving equation (17) for Π and substituting the result into equations (30)–(33), we obtain the optimal ratios of heat exchanger areas to total heat transfer area in terms of the coefficient of performance ψ:

$$\frac{A_g}{A}=\sqrt{\frac{U_h}{U_g}\frac{\psi T_l+(1-C)(1-\psi)T_p}{\psi T_l+C^2\psi(\psi-1)T_h+(1-C)^2(1-\psi)T_p}}, \tag{35}$$

$$\frac{A_a}{A}=\sqrt{\frac{U_h}{U_{ac}}\frac{\psi T_l+(1-C)(1-\psi)T_p}{\psi T_l+C^2\psi(\psi-1)T_h+(1-C)^2(1-\psi)T_p}}, \tag{36}$$

$$\frac{A_c}{A}=\sqrt{\frac{U_h}{U_{ac}}\frac{(\psi-1)[C\psi T_h+(1-C)T_p]}{\psi T_l+C^2\psi(\psi-1)T_h+(1-C)^2(1-\psi)T_p}}, \tag{37}$$

$$\frac{A_e}{A}=\sqrt{\frac{U_h}{U_e}\frac{(\psi-1)[C\psi T_h+(1-C)T_p]}{\psi T_l+C^2\psi(\psi-1)T_h+(1-C)^2(1-\psi)T_p}}. \tag{38}$$

These relationships are illustrated in Fig. 6(a). It is seen clearly that for a given total heat transfer

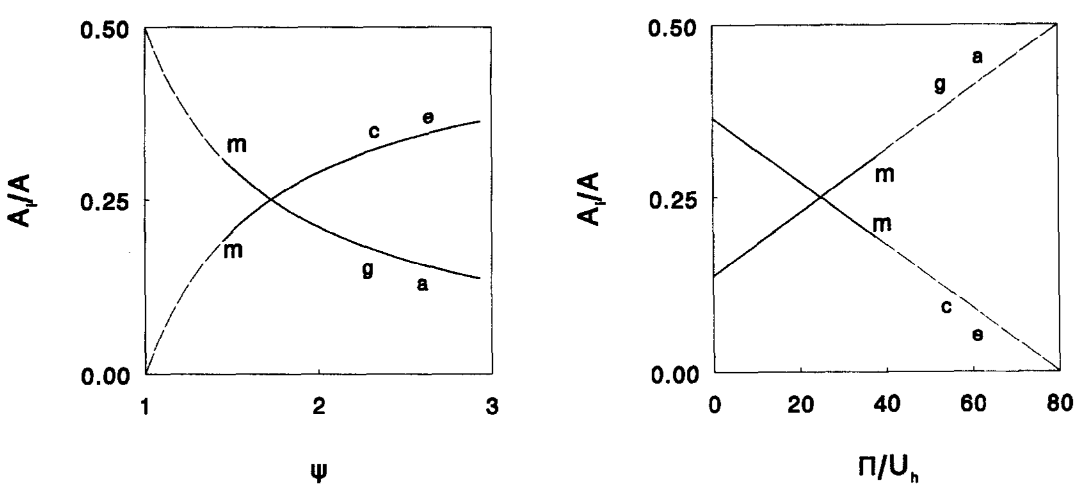

Fig. 6. The curves of the ratios of the heat transfer areas A_i/A (i = g, a, c, e) vs (a) the coefficient of performance ψ and (b) the ratio of specific heating load to the heat transfer coefficient, Π/U_h, for equal heat transfer coefficients, $U_g = U_{ac} = U_e$. The non-optimal regions are shown in dashed lines. The curves have been drawn for the temperatures $T_h = 130°C$, $T_p = 50°C$ and $T_l = 20°C$.

area, A, the heat transfer areas associated with the heat pump, A_c and A_e of the condenser and the evaporator, should be increased as the coefficient of performance ψ increases, while the heat transfer areas associated with the heat engine, A_g and A_a of the generator and absorber, should be correspondingly decreased. The reason clearly is that, as the coefficient of performance of the system increases, a larger fraction of the delivered heat q_p comes from the heat pump rather than from the inefficiency of the heat engine, thus requiring larger areas of transfer.

Using equations (17) and (30)–(33), the reader may obtain the optimal relations between A_i/A and Π. The curves of A_i/A vs Π/U_h are presented in Fig. 6(b). It is interesting to note that for a given A, the relations between A_i and Π/U_h are linear.

According to equation (24) and Fig. 6, we find the optimal regions of A_i:

$$A_g \leqslant (A_g)_m, \tag{39}$$

$$A_a \leqslant (A_a)_m, \tag{40}$$

$$A_c \geqslant (A_c)_m, \tag{41}$$

$$A_e \geqslant (A_e)_m, \tag{42}$$

where $(A_i)_m$ are, respectively, the optimal heat transfer areas of the four heat exchangers when $\psi = \psi_m$. The points $(A_i)_m$ are marked on Fig. 6 with an 'm', and the non-optimal regions shown in dashed lines. Substituting equation (24) into equations (35)–(38), the reader may obtain the expressions of $(A_i)_m$. When $U_h = U_h = U_p$, they may be simply written as

$$(A_g)_m = A\sqrt{\frac{U_h}{U_g}}\frac{\sqrt{T_hT_p} - T_l}{T_h - T_l}, \tag{43}$$

$$(A_a)_m = A\sqrt{\frac{U_h}{U_{ac}}}\frac{\sqrt{T_hT_p} - T_l}{T_h - T_l}, \tag{44}$$

$$(A_c)_m = A\sqrt{\frac{U_h}{U_{ac}}}\frac{T_h - \sqrt{T_hT_p}}{T_h - T_l}, \tag{45}$$

$$(A_e)_m = A\sqrt{\frac{U_h}{U_e}}\frac{T_h - \sqrt{T_hT_p}}{T_h - T_l}. \tag{46}$$

6. CONCLUSION

In the design of absorption heat pump systems a detailed knowledge of the primary performance parameters, such as the coefficient of performance, the specific heating load and the heat transfer areas of the heat exchangers, is highly desirable. The results derived from the cycle model of a heat-engine-driven heat pump can determine the optimal working regions of the primary performance parameters of an endoreversible absorption heat pump. They can guide the evaluation of existing real absorption heat pumps and influence the design of future absorption heat pumps.

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