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OPTIMAL CONTROL CONFIGURATION OF HEATING AND HUMIDIFICATION PROCESSES PART II

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In partea I a acestei lucrări am investigat cum energia apei calde poate să fie folosită pentru a încalzi şi umidifica în acelasi timp în sezonul rece. Am stabilit că variabila pentru optimizare lătimea umidificatorului/incalzitor care sa ofere transferul de masă şi caldura între apa şi aer cu producerea de entropie minimă.

Modelul matematic stabilit în partea I a prezentei serii este acum folosit pentru exemplificarea proceselor prin soluţii numerice. Pentru valori date ale temperaturii şi umidităţii relative a aerului la intrarea în umidificator precum şi pentru valori dorite ale aerului la iesirea din umidificator, determinam temperatura apei necesare pentru umidificare (la intrarea şi ieşirea din dispozitiv) precum şi cantitatea consumată.

In the part I of this paper we investigated how the energy carried by warm water can be best used for heating the outdoor cold air while increasing its humidity in cold seasons. We set to check the optimal width of a humidifier/heater that will offer the transfer of mass and energy between air and water with the minimum of entropy production.

The mathematical model established in the part I of the present series is now used for introducing numerical examples. For given values of the air temperature and relative humidity at the humidifier inlet, as well as for the desired air temperature and relative humidity at the device outlet, we determine the temperatures (inlet/outlet) of the water for humidification and the quantity to be consumed.

Keywords: optimal control theory, minimum entropy production, air humidification

Nomenclator

Latin symbols

A = area of water surface in contact with air
B = atmospheric pressure
$C_1 \dots C_5$ = empirical coefficients
C_v = heat capacity
d = number of controls
$\vec{d}$ = vector of controls for the process

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$\vec{F}$ = vector of evolution expressions for $\vec{n}$
$G(\vec{n}_1, t_1)$ = desired combination of state functions at the final time
G = vaporization rate of water
H = Hamiltonian of optimization
ΔH = enthalpy of water vapor
I = cost (object) functional for the process
k = coefficient of heat transfer
L = rate of dissipation
M = molar mass
m_v = mass of water vapor per volume of air
m_w = mass of water in the water layer per area of surface of water-air contact
n = number of state functions
$\vec{n}$ = vector of state functions of the system
p = partial pressure of water
Q = rate of heat transfer
R = ideal gas constant
ΔS = entropy change (production) during the process
T = temperature
t = time
v = air velocity over water surface
V = volume
x = position in flow direction
y = width of channel
z = height of air channel

Superscripts
* = optimal value;
$\dot{s}$ = ds/dt

Subscripts
$_0$ = initial value
$_1$ = final value
$_a$ = related to air
$_{sat}$ = at saturation
$_v$ = related to vapor
$_w$ = related to water

Greek symbols
λ_l = latent heat of water
$\vec{\psi}$ = vector of co-state (adjoint) variables
$\vec{\omega}$ = vector of state variables T_a, T_w, m_v, m_w
φ = relative humidity of air

1. Introduction

In a previous paper we were undertaking an analysis of the process of heating and humidification of moist air using optimal control theory as the tool of the analysis.

The objective of the analysis was to obtain the best configuration of a humidification and heat transfer process consistent with a given set of constraints and a specific operating goal. The current state of knowledge is reflected in [1], [2], [3], [4], [5].

We will briefly remember the state variables defined in order to solve the optimal control problem:

1. $\boldsymbol{T_a(x)}$ = the air temperature, [K] (state variable for the air side)
2. $\boldsymbol{T_w(x)}$ = the water layer temperature, [K] (state variable for the water side)
3. $\boldsymbol{m_v(x)}$ = mass of water vapor per volume of air [kg/m^3] (state variable for the air side)
4. $\boldsymbol{m_w(x)}$ = mass of water in the water layer per area of surface of water-air contact [kg/m^2] (state variable for the water side)

We considered the temperatures of the air and water layer to be uniform throughout the two media at each value of x.

The optimal control theory has been applied for various thermodynamics problems [6-14].

2. Solution of the optimal control problem

Solving the problem implied finding solutions to the set of equations for the state variables among which there were the optimal solutions for the chosen control.

Let us summarize the canonical equations for the state variables as functions of x used to compose the Hamiltonian:

$$\frac{dT_a}{dx} = \frac{k}{C_4 C_{p,vol}\, z}\, y\,(T_w - T_a) \tag{2.1}$$

$$\frac{dT_w}{dx} = \frac{yk(T_w - T_a) + \lambda_l C_1 \left(1 - \frac{m_v R\, T_a}{M_w C_3 e^{-\lambda_l/RT_a}}\right)(y + c_2\, c_4)}{c_{p,w}\, c_5 \cdot m_w} \tag{2.2}$$

$$\frac{dm_v}{dx} = \frac{C_1}{z}\left(1 - \frac{m_v R\, T_a}{M_w C_3 e^{-\lambda_l/RT_a}}\right)\left(\frac{y}{C_4} + C_2\right) \tag{2.3}$$

$$\frac{dm_w}{dx} = \frac{C_1}{C_5}\left(1 - \frac{m_v R\, T_a}{M_w C_3 e^{-\lambda_l/RT_a}}\right)(y + C_2\, C_4) \tag{2.4}$$

We considered the vector space $\vec{\omega}$ of the state variables $T_a(x)$, $T_w(x)$, $m_v(x)$, $m_w(x)$ and formed the Hamiltonian

$$H(\vec{\omega}, y, \vec{\psi}) = I(\vec{\omega}, y) + \vec{\psi} \cdot F(\vec{\omega}, y) \tag{2.5}$$

with $\vec{\psi}$ being the co-state vector.

The optimal solution to the problem was the boundary solution $y^* = y_{max}$ over the entire length of the heater/humidifier.

The equations were solved with inlet/outlet boundary conditions on air temperature and humidity. The other parameters would follow in the solution from these parameters.

3. Numerical analysis of the obtained optimal solution

In order to illustrate the bang-bang solution obtained by solving the canonical equations reminded in paragraph 2 we performed an analysis on a numerical example. In the table below values attributed to all constants that appear in the equations are given.

Table 3.1

Numerical values for equation constants

No.	Constant	Value	Unit
1	C_1	2.57	Kg/m^{2}s
2	C_2	0.883	s/m
3	C_3	$1.166 \cdot 10^{11}$	Pa
4	C_4	1	m^{2}s
5	C_5	1	m^{2}s
6	R	8.31447	J/molK
7	$c_{p,a}$	1302	J/m^{3}K
8	$c_{p,w}$	4181.3	J/kgK
9	M_w	0.018015	Kg/mol
10	λ	2.405·106	J/kg
11	k	13	W/ m^{2}K

The graphs below illustrate the bang-bang solution for the air temperature T_a (red), water temperature T_w (black), air humidity contents m_v (green) liquid water in the stream m_w (blue), and relative humidity rh of the air (cyan).

In the figs 3.1 – 3.4 below the evolution of the parameters, $T_{w\ in}$, $T_{w\ out}$ $m_{w\ in}$ and $m_{w\ out}$ is also shown depending on the position x, when the set of the other 4 parameters is given: $T_{a\ in}$ and $T_{a\ out}$, $m_{v\ in}$ and $m_{v\ out}$. The parameter $rh = \varphi$ is not a new independent parameter but related to the humidity contents as it is defined as the ratio of the vapor mass in an air volume and the maximum vapor mass in the same volume, i.e. saturation, at the same temperature,

$$\varphi = \frac{m_v}{m_s} \tag{3.1}$$

The reason for also showing the relative humidity in addition to humidity contents in the graphs is determined by the practical engineering use of this information when referring to the humidification process. The outdoor air temperature T_a and relative humidity are always known and must be taken into account in the design calculations. Likewise one strives to achieve a certain temperature with a certain relative humidity at the outlet of the humidification device, i.e. indoors. These boundary conditions determine the rest of the parameters that are of interest for the design engineer: the water temperature to be used for humidification, its temperature at the outlet of the device (should further use of it be found), as well as the quantity of water required by the process.

The system of equations is solved setting the boundary conditions for 4 of the parameters, 2 of each are set for the humidification device inlet at the position $x = 0$ m and the rest of 2 for the humidification device outlet at the position $x = 10$ m (arbitrarily selected).

Every diagram figure is accompanied by a table with the values used to draw the curves. The graphs were obtained using MATLAB software application.

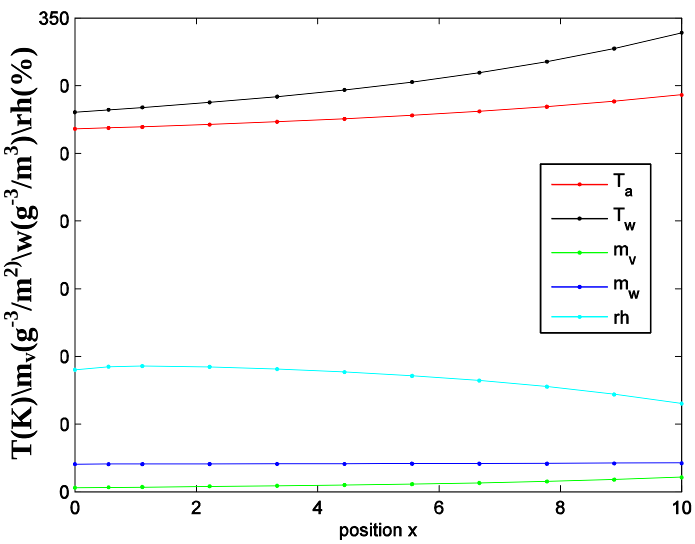

Fig. 3.1 Variation of the state variables when the air increases its temperature T_a from -50 C with r_{hout}= 90% to 200 C with r_{hin}= 65%.

The air temperature and relative humidity at the inlet respective outlet of the humidification coil considered will bear the subscript $_{in}$ for indoor parameters, respective $_{out}$ for outdoor parameters – convention established due to the air/water side.countercurrent flow which determines different numbers for the same state on The outdoor parameters are for the outdoor air that enters the humidification coil and the indoor parameters are considered for the indoor air that is delivered to the conditioned space.

Table 3.2

$T_{a\ out}$= -5^0 C rh_{out}= 90% $T_{a\ out}$= 20^0 C rh_{in} =65%

x[m]	T_a[K]	T_w[K]	m_v[g/m^2]	m_w[g/m^3]	rh[%]
0	268.0000	280.3454	3.1	20.3	90
10	293.0000	339.0010	10.6	21.1	65

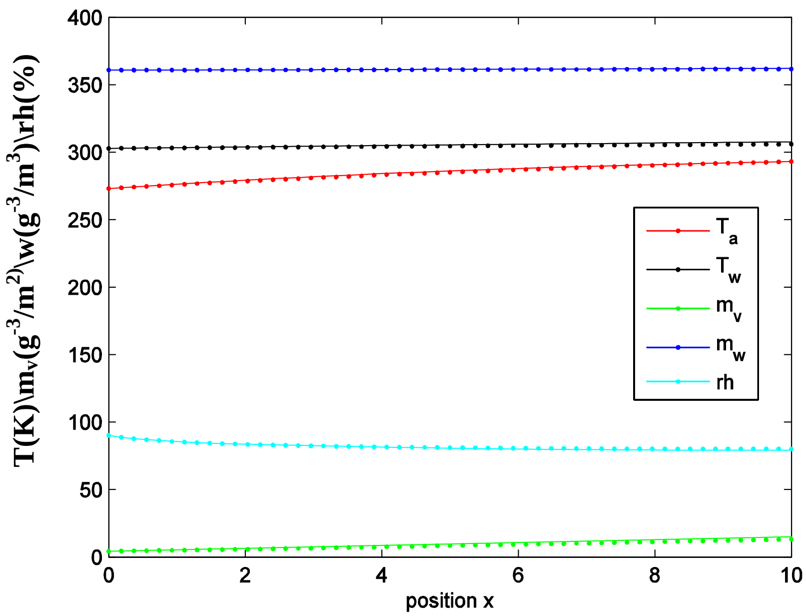

Fig. 3.2 Variation of the state variables when the air increases its temperature T_a from 0^0 C with rh_{out}= 90% to 20^0 C with rh_{in} =80%

Table 3.3

$T_{a\ out}$= 0^0 C rh_{out}= 90% $T_{a\ out}$= 20^0 C rh_{in} =80%

x[m]	T_a[K]	T_w[K]	m_v[g^{-3}/m^2]	m_w[g^{-3}/m^3]	rh[%]
0	273.0000	302.7772	4.3	360.7	90
10	293.0000	305.8978	13.0	361.6	80

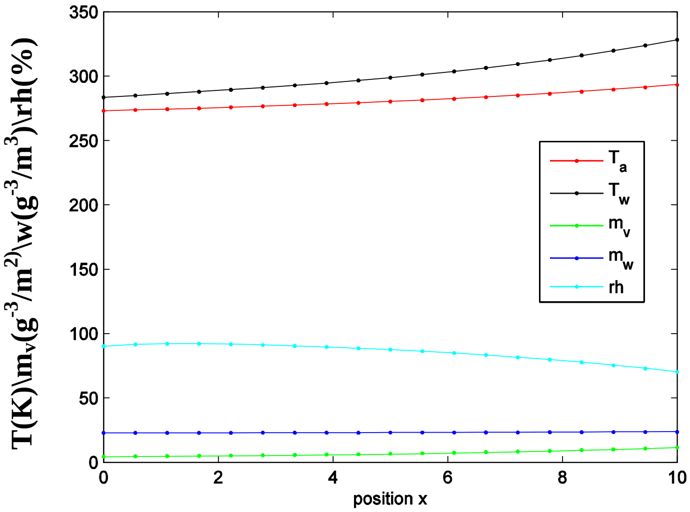

Fig. 3.3 Variation of the state variables when the air increases its temperature T_a from 0^0 C with rh_{out}= 90% to 20^0 C with rh_{in} =70%.

Table 3.4

$T_{a\ out}$= 0^0 C rh_{out}= 90% $T_{a\ out}$= 20^0 C rh_{in} =70%

x[m]	T_a[K]	T_w[K]	m_v[g/m^2]	m_w[g/m^3]	rh[%]
0	273.0000	283.3478	4.3	22.7	90
10	293.0000	328.1186	11.4	23.4	70

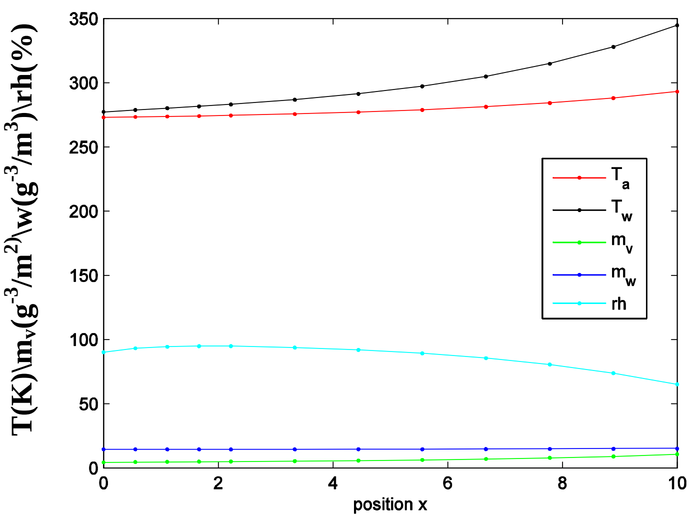

Fig. 3.4 Variation of the state variables when the air increases its temperature T_a from 0^0 C with rh_{out}= 90% to 20^0 C with rh_{in} = 65%.

Table 3.5

$T_{a\ out}$= 0^0 C rh_{out}= 90% $T_{a\ out}$= 20^0 C rh_{in} =65%

x[m]	T_a[K]	T_w[K]	m_v[g/m^2]	m_w[g/m^3]	rh[%]
0	273.0000	277.2584	4.3	14.3	90
10	293.0000	344.6344	10.6	15.0	65

4. Conclusions

There are various instances when indoor spaces could be heated and humidified in the same time through the interaction between warm water and outside cold air depending on the maximum level of relative humidity accepted by the respective spaces. That can save on the energy spent for humidification in addition to that for heating.

The numerical results of the optimization problem show that for temperatures of water sufficiently high the cold outdoor air can be heated up to 200C while increasing the relative humidity of the air to admissible values for the indoor conditions. The equations are written for water irrespective of its temperature – below 0^0C or above 100^0C - in order to generalize the problem. However, it is the hot water and not the steam that is of interest in the matter at hand as water already puts energy into becoming steam and the aim is to minimize the entropy generation.

Some authors [19] mention humidifying with heating as well, representing the process in the psychrometric (enthalpy – humidity contents) diagram, without close investigation into a range of values for the water temperature.

In a previous paper we concluded that using optimal control theory it was possible to solve the system of 4 equations with 8 variables by fixing 4 of the variables at the boundaries and obtaining the remaining 4 from the optimization.

If, however, we need to fix 5 variable values out of the 8, e.g. if in addition to the air quality outside and inside we also want to set the temperature of the entering humidification water stream, we need a fifth degree of freedom. That could e.g. be the length of the humidifier which currently is set to L = 10 m.

Alternatively, we may summarize the results presented above in a diagram which can serve the purpose just mentioned. The graph shows the temperature of the incoming water (blue) and discharged water (green) needed in order to obtain a certain relative humidity and temperature at the humidification outlet, for reference the inside target air temperature (red) is also shown. Read from the ordinate to the abscissa tells us which inside air humidity can be achieved with a given supply water temperature. A different humidifier length will of course change this picture.

The curves in the diagram below were calculated with the following set of data: $T_{a\ in} = 0^0$C, $T_{a\ out} = 20^0$C, $rh_{in} = 90\%$.

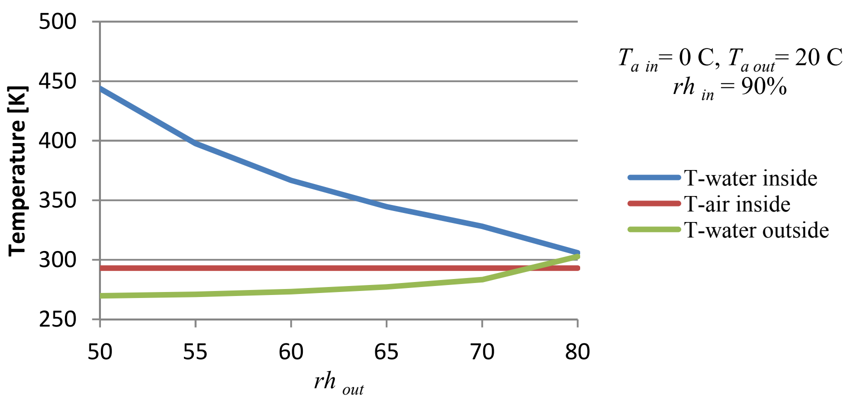

Fig. 4.1 Variation of the relative humidity at the outlet with the temperature of the water for humidification

Note that low relative humidity at the outlet of the humidification device requires high water temperature but in a small amount while the temperature of the water needed for humidification decreases rapidly as the relative humidity at the outlet increases.

The same equations equally apply to summer conditions for evaporative cooling. This is already hinted at in several of the low humidity graphs where the water discharge temperature at $x = 0$ (black) is below that of the incoming air (red), i.e. we see a distinct evaporative cooling even against air at relative humidity of 90%.

We envisage further applications of optimal control for validating design of heating/cooling/humidification by applying other control functions or having different objective functions like water consumption/temperature, loss of availability, etc.

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