Effects of heat leak on the performance characteristics of Carnot like heat engines and heat pumps

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Abstract

Effects of heat leakage on the performance characteristics of Carnot like heat engine and heat pump models are studied. The heat leakage term is assumed to be temperature dependent with non-Newtonian heat transfer law. Thus the heat leakage term is proportional to the temperature difference with exponent m. Values of m are considered in the range 0-2. It is shown that the power output of the heat engine is not affected due to the assumptions of the model, but the main effect is reducing the efficiency of the system. Correspondingly, considering heat pumps, the cooling rate and coefficient of performance (cop) are reduced due to heat leakage. The cooling is not a monotonic function of cop. This was not observed in previous studies with the assumption of a constant heat leakage term. Finally, characteristic curves are shown for both heat engines (power vs. efficiency curves) and for heat pumps (cooling rate vs. coefficient of performance curves).

Keywords: Heat engines, heat pumps, endoreversible, heat leakage, temperature dependent, cooling rate.

Resumen

Se estudian los efectos de la fuga de calor en las características de rendimiento de Carnot como motor térmico y los modelos de bomba de calor. El término de fugas de calor se supone que es dependiente de la temperatura con la ley de transferencia de calor no-newtoniano, de ahí el término de fuga de calor es proporcional a la diferencia de temperatura con el exponente m. Los valores de m se consideran en el rango de 0-2. Se muestra que la potencia del motor térmico no se ve afectada debido a la asunción del modelo, pero el efecto principal es la reducción de la eficiencia del sistema. Por otra parte, mientras que las bombas de calor teniendo en cuenta, la velocidad de enfriamiento y el coeficiente de rendimiento (COP) se reducen debido al calor de fuga. La refrigeración no es función monótona de la policía. Esto no fue observado en estudios previos con la suposición de un término de fuga de calor constante. Por último, se muestran las curvas características de ambos motores de calor (las curvas de eficiencia energética) y las bombas de calor (velocidad de enfriamiento vs coeficiente de curvas de rendimiento).

Palabras clave: Calentar motores, bombas de calor, las fugas endoreversible, el calor, dependiente de la temperatura, velocidad de enfriamiento.

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I. INTRODUCTION

In this article, Carnot like [1, 2] heat engine and heat pump models are considered under the constraints of finite heat transfer rates and temperature dependent heat leakage. The effect of heat leakage on the performance of heat engines and heat pumps has been considered by several studies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The heat leakage term was assumed to be constant, where heat leaks directly from the hot reservoir to the cold reservoir.

Finite-time thermodynamic analysis of the endoreversible heat engines [3, 4, 5] predicted unchanged maximum power output but at lower efficiency, while the maximum efficiency was given by the Carnot efficiency at zero power production. In reality, heat engines, made up of several moving parts and different materials suffer from realistic limitations. In order to demonstrate the performance of real heat engines, generalized power versus efficiency characteristics of heat engines were produced by considering the thermoelectric generator as an illustrative example [6]. Different power producing systems suffer from different types of irreversibility mechanisms. The key irreversibility sources include fluid friction (Brayton cycle), the constraint of the equation of state of the engine's working fluid (Rankine cycle), and heat leak (the thermoelectric generator) [7]. The effect of heat resistance and heat leakage on the optimal performance of finite-time heat engines was investigated based on a generalized heat transfer law $q \propto \Delta(T^n)$ [8].

Also heat pumps were originally treated with finite heat transfer rate as the sole dissipation mechanism [9, 10, 11, 12, 13]. Later reciprocating chillers [14] were analyzed and their characteristics compared to experimental data. The effect of heat leak on the performance of heat pumps was also addressed by [15, 16, 17] and their characteristic flows described using power-degradation or power-coefficient of performance coordinates.

In a different study [18], the maximum power production of an ideal Otto cycle was considered using optimal control theory methods to predict the paths leading to maximum power (or maximum efficiency for a given amount of heat input). The model considered here included heat leakage to the cylinder wall which depends on the temperature on the working fluid inside the cylinder.

The argument in the following paragraph aims to highlight the physics behind the temperature dependent heat leakage assumption.

Heat transfer theory deals with finding heat transfer coefficients based on experiments. The heat transfer coefficient is calculated from the Nusselt number, hL/k, (after Wilhelm Nusselt, who made significant contributions to the theory of heat convection heat transfer), where h is the heat transfer coefficient for convection, L is a characteristic length of the set up considered, and k is the thermal conductivity of the fluid. The Nusselt number is proportional to the Prandtl number Pr. The Prandtl number is the parameter that relates the relative thicknesses of hydrodynamic and thermal boundary layers. The kinematic viscosity of fluid conveys information about the rate at which momentum may diffuse through the fluid because of molecular motion. The thermal diffusivity tells us the same thing in regard to the diffusion of heat in the fluid. Thus the relation of these quantities expresses the relative magnitudes of diffusion of momentum and of heat in the fluid [19]. Both Prandtl number and thermal conductivity depend on temperature [19].

The article is arranged as follows: In section II, the heat engine model is described, in section III, the heat pump model is developed based on the results of the heat engine model given in section II, and finally a summary and conclusions are given in section IV.

II. HEAT ENGINE MODEL

In this section an endoreversible heat engine model is considered with the assumptions of finite heat transfer rate that follows a Newtonian heat transfer law and of a heat leakage term. The heat leakage term follows a non-Newtonian heat transfer law. This assumption is based on the observations and results of convection heat transfer theory [19]. The essential points are reviewed here for convenience of the reader.

As mentioned in section I, the heat transfer coefficient is calculated based on Nusselt number. The traditional expression for calculation of heat transfer in fully developed flow in smooth tubes is

$$Nu_d = 0.023\,\mathrm{Re}_d^{0.8}\,\mathrm{Pr}^n$$

The properties in this equation are evaluated at the fluid bulk temperature, and the exponent n has the following values:

$n = 0.4$ for heating of the fluid.
$n = 0.3$ for cooling of the fluid.

The bulk temperature is representative of the total energy of the flow at the particular location.

Simplified equations for free convection from vertical surfaces to air at atmospheric pressure can be found in table 7-2, p. 334 of ref. [19].

From these equations one observes that the heat transfer coefficient is proportional to the temperature difference to the power m where m=1/4 for laminar and m=1/3 for turbulent flow. Further, properties of air at atmospheric pressure are given in the Table A-5, page 602 in the appendix of ref. [19].

It is clear from that table that the Prandtl number is approximately constant. But the thermal conductivity changes drastically with temperature. E.g., the thermal conductivity almost doubles over the range 300-600K. The table shows that the conductivity is approximately linear with T. Fig. 1 displays the thermal conductivity, Prandtl number, and the ratio of thermal conductivity to Prandtl number to the exponent 1/3. The figure shows that the Prandtl number is approximately constant while the ratio of thermal conductivity to Prandtl number to the third power depends highly on temperature. These observations affect heat transfer rates across the boundaries of heat engine and heat pump models.

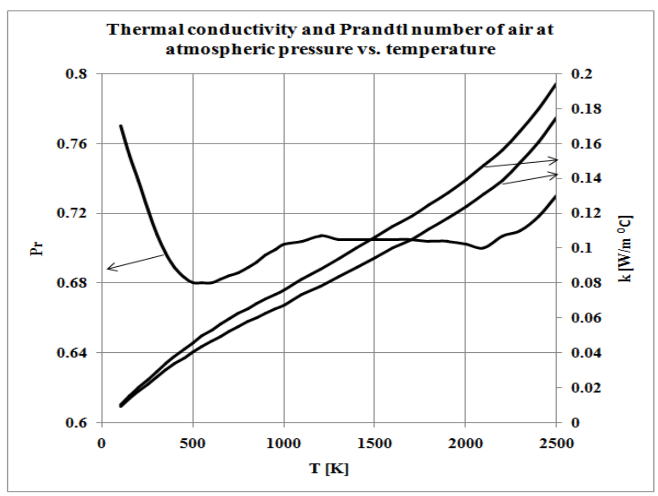

FIGURE 1. Prandtl number, thermal conductivity, and the ratio of thermal conductivity to Prandtl number to the power one third for air at atmospheric pressure are shown as functions of temperature in the range 100-2550K. While the Prandtl number is approximately constant, the thermal conductivity and its ratio to the Prandtl number to the third power are strongly dependent on temperature.

The schematic of the endoreversible heat engine is given in Fig. 2. The heat engine works between two heat reservoirs, a hot reservoir at T_H and a cold reservoir at T_L, respectively. Due to heat resistance, the temperature at the hot side of the engine is reduced to T_h and the temperature at the cold side is raised to T_c. Heat leaks from the hot engine side to the cold reservoir. It is assumed here that the heat leaks at a mean temperature value (an arithmetic average of the hot engine side temperatures). This assumption is based on physical observations described earlier in this article. We believe that it describes the real processes involved more accurately than the assumption of a constant term.

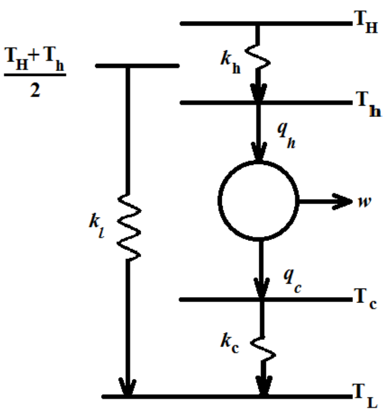

FIGURE 2. Schematic of the endoreversible heat engine with a heat leakage. Heat leaks from the hot side to the cold side at a moderate temperature. This assumption introduces a temperature dependent heat leakage term to the heat engine.

A. Endoreversible heat engine

The endoreversible heat engine model equations are reviewed next.

The rate of heat input (q_h) to the endoreversible heat engine follows a Newtonian heat transfer law and is given by

$$q_h = k_h (T_H - T_h). \tag{1}$$

The symbol k_h is the heat conductance at the hot side of the heat engine.

Similarly, the rate of heat rejection (q_c) from the endoreversible heat engine is given by

$$q_c = k_c (T_c - T_L). \tag{2}$$

The symbol k_c is the heat conductance at the cold side of the heat engine.

The endoreversibility constraint implies that the internal entropy generation is assumed to be zero. This constraint could be written as follows:

$$\frac{q_h}{T_h} = \frac{q_c}{T_c}. \tag{3}$$

The net power output is found from the first law of thermodynamics and is given by

$$w = q_h - q_c. \tag{4}$$

And finally, by definition the thermal efficiency (η_0) is calculated as

$$\eta_0 = \frac{w}{q_h} \tag{5}$$

These equations are converted to dimensionless form by dividing the heat transfer rates by the quantity $k_h T_H$ and after some mathematical manipulation are re-written as the dimensionless rate of heat input:

$$q_h^* = \frac{q_h}{k_h T_h} = \frac{(1-\eta_0-\tau)}{(1-\eta_0)\left(1+\frac{1}{k}\right)}. \tag{6}$$

The symbol τ is the ratio between the reservoir temperatures ($\tau = T_L/T_H$) , and k is the ratio of the heat conductances ($k = k_c/k_h$).

The dimensionless rate of heat rejection is correspondingly

$$q_c^* = \frac{q_h}{k_h T_h} = q_h^* (1-\eta_0). \tag{7}$$

And finally, the dimensionless net power output is given by

$$p = \frac{w}{k_k T_h} = \eta_0 q_h^*. \tag{8}$$

B. Introducing heat leakage

As described in the introduction, the rate of heat leakage (q_l) is often assumed to be constant:

$$q_l = k_l (T_H - T_L). \tag{9}$$

The symbol k_l is the heat leakage conductance.

However, in this article the rate of heat leakage is assumed to be temperature dependent and given by

$$q_l = k_l \left[\left(\frac{T_H + T_h}{2} \right) - T_L \right]^m. \tag{10}$$

The symbol m is the exponent of the temperature difference and its value could range between 0 and 2 (as considered in this study).

The total rate of heat input to the heat engine is then

$$Q_h = q_h + q_l. \tag{11}$$

Similarly, the total rate of heat rejection is

$$Q_c = q_c + q_l. \tag{12}$$

With this assumption, the power output is not changed and is given as before by Eqs. (4) and (8), but the thermal efficiency (η) of the heat engine is now

$$\eta = \frac{w}{Q_h}. \tag{13}$$

The heat leak term is re-written in dimensionless form as

$$q_l^* = \frac{q_l}{k_h T_h} = k_{leak}\left(1 - q_h^*/2 - \tau\right)^m. \tag{14}$$

The symbol k_{leak} is the dimensionless heat conductance coefficient $\left(k_{leak} = {k_l T_h^{m-1}}\big/{k_h}\right)$.

Based on these equations, the power – efficiency characteristic curves are presented in Figs. 3 and 4. Fig. 3 shows the power output relative to its maximum value vs. efficiency for different values of the heat leak coefficient. The plots are made for m = 1.25. The figure demonstrates the degradation of the efficiency as the heat leak increases. The heat leak with temperature dependence has a similar effect as the constant heat leak but with a smaller reduction in efficiency.

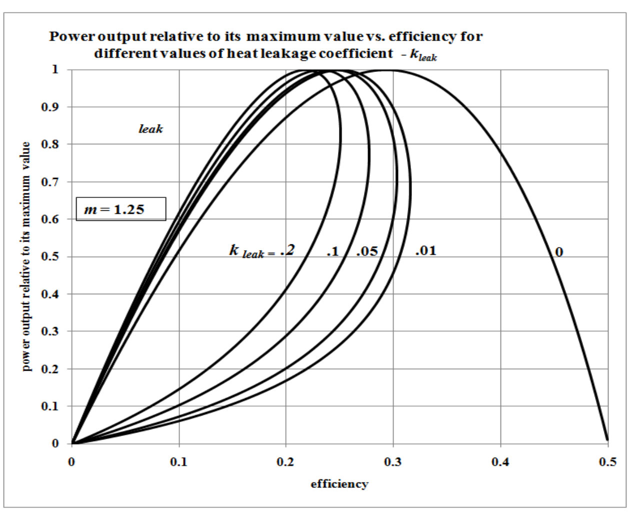

FIGURE 3. Power output relative to its maximum value *vs.* efficiency for different values of the heat leak coefficient. While the maximum power point is not changed, the efficiency reaches a maximum and depends strongly on heat leakage.

Fig. 4 shows similar plots of power vs. efficiency with the exponent as a parameter while the heat leak coefficient is fixed. From the figure one observes that the smaller the exponent, the larger the reduction in efficiency. The case of m=0 represents a constant heat leakage model which results in higher energy losses and thus lower maximal efficiency.

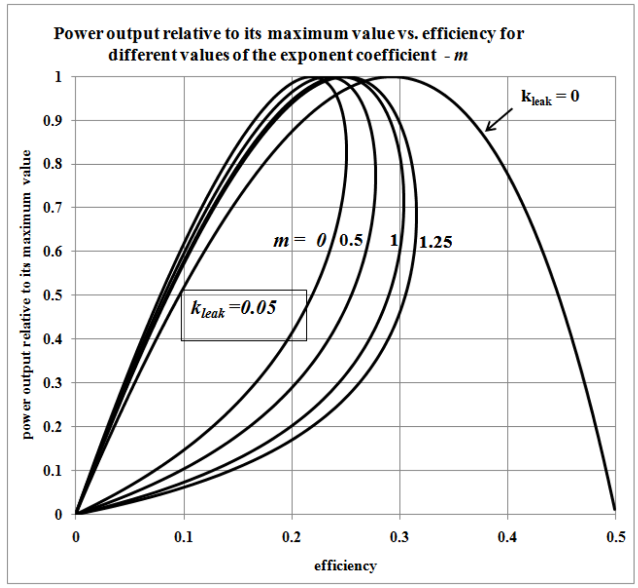

FIGURE 4. Power output relative to its maximum value *vs.* efficiency for different values of the exponent *m*. While the maximum power point is not changed, the efficiency reaches a maximum and depends strongly on the exponent. The higher the value of the exponent, the smaller the heat leakage observed.

III. HEAT PUMP MODEL

Heat pumps work in reverse of heat engines. Fig. 5 is the schematic of the heat pump model. The home refrigerator is a good example to clarify the parameters of the heat pump model. As depicted in Fig. 5, heat leaks from an average temperature on the hot side to the inner space of the refrigerator. The hot reservoir is the surroundings of the kitchen at temperature T_H and the inside of the refrigerator is the cold space at temperature T_c. The working fluid inside the tubes is at lower temperature T_L and after passing the compressor the working fluid reaches a higher temperature T_h. Some of the heat rejected from the working fluid at the back of the refrigerator to the surroundings then by processes of heat convection and of heat conduction leaks back to the interior space.

By reversing flows, the equations of the heat pump model can be summarized immediately from the results of the heat engine model with careful reversal of signs and picking appropriate working parameter ranges.

The dimensionless rate of heat rejected from the endoreversible heat pump at the hot side is then

$$q_h^* = \frac{\eta_0 + \tau - 1}{(1-\eta_0)\left(1 + \frac{1}{k}\right)}. \tag{15}$$

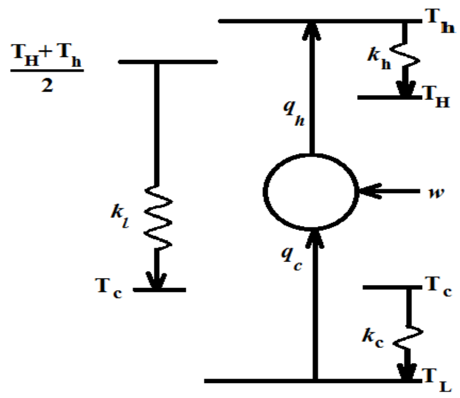

FIGURE 5. Schematic of the endoreversible heat pump with heat leakage. Heat leaks from the hot side to the cold side at a moderate temperature. This assumption introduces a temperature dependent heat leakage term to the heat pump.

The rate of heat input from the endoreversible heat pump at the cold side is given by Eq. (7). The heat leak term for the heat pump model is correspondingly given by Eq. (10). Adding the heat leak term will give the total amount of heat input and heat rejection to the heat pump and are given by Eqs. (11) and (12), respectively.

The power input to the system is given as before by Eqs. (4) and (8). In common practice, heat pump performance is reported in terms of the coefficient of performance which is defined as the ratio of the heat absorbed by the machine and the power input (cooling systems) or the ratio of the heat rejected and the power input (heating systems). The coefficient of performance of cooling systems (cop) is thus

$$cop_L = \frac{Q_c}{w}. \tag{16}$$

Figs. 6 and 7 depict the characteristic curves (cooling rate vs. coefficient of performance) of the heat pump model. While the ideal system without heat leak (k_{leak}=0) has a monotonic tradeoff between cooling rate and cop, we see that the coefficient of performance reaches a maximum value (lower than the Carnot limit) for the leaky system, depending on both the heat leak coefficient and the exponent *m*. This general behavior was observed for the constant heat leak model as well. On the other hand, only the constant heat leak model (*m*=0) shows a monotonic increase of cooling rate at small cop, i.e. at high rates of operation. Temperature dependent heat leak results in a more realistic maximum cooling rate, a feature that was not observed before.

IV. SUMMARY AND CONCLUSIONS

In this article we considered irreversible models of heat engines and heat pumps. The irreversibilities account for non-vanishing heat transfer rates and temperature dependent heat leakage. The temperature dependence of the heat leakage is based on the observations and correlations from convection heat transfer theory. The convection heat transfer depends on the Prandtl number in thermal conductivity of the working fluid. The thermal conductivity depends on temperature as was observed in the literature [19]. The heat leakage term, based on the assumption of the models introduced in sections II and III, does not affect the power output of the heat engines. On the other hand, efficiency depends strongly on the heat leak coefficient. By comparing the models of constant heat leak and temperature dependent heat leak terms, one observes that both models predict the same type of characteristic curves (power vs. efficiency curves). The differences are in the absolute amounts of heat leak, where the constant heat leak model is more severe than the temperature dependent heat leak model.

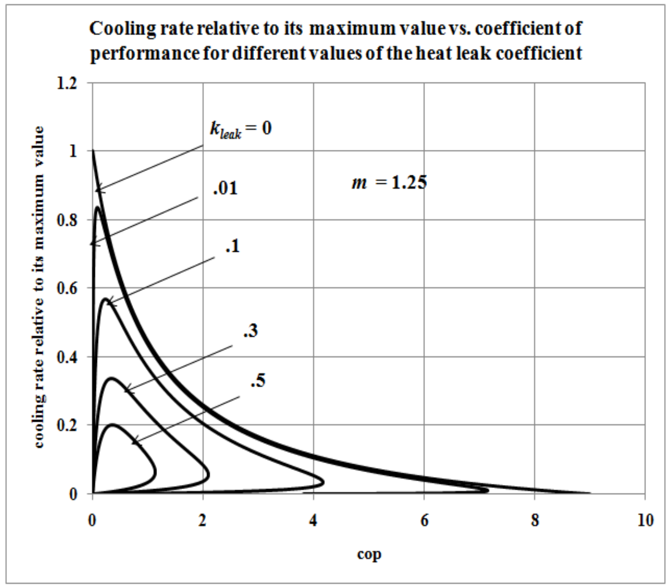

FIGURE 6. Cooling rate relative to its maximum value vs. coefficient of performance (cop) for different values of the heat leak coefficient. Both characteristics, cooling rate and coefficient of performance, reach a maximum and depend strongly on heat leakage.

Similar results are observed for heat pump systems, regarding the coefficient of performance. The major difference between the two model assumptions observed in this study, is the effect of heat leakage on the cooling rate of the heat pump. Although we focused on cooling rates in this study, heating rates behave in a similar way. As was observed from the constant heat leak term, the cooling rate has no maximum point. On the other hand, the cooling rate does reach a maximum point assuming a temperature dependent heat leak term.

The temperature dependent heat leak assumption, as it rests on a physical basis, decisively affects the behavior of heat engines and heat pumps.

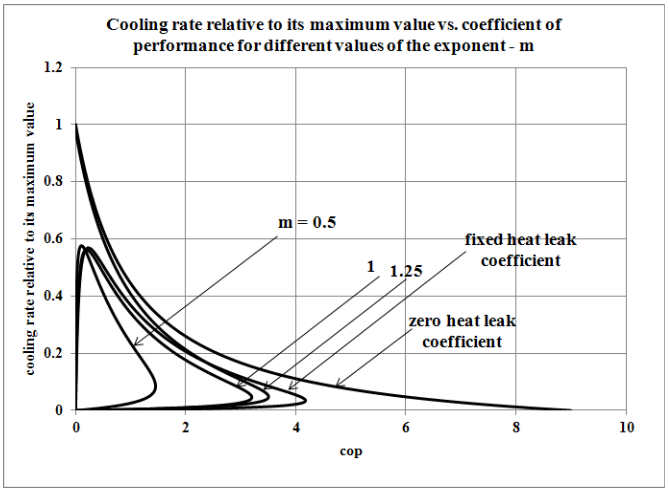

FIGURE 7. Cooling rate relative to its maximum value vs. coefficient of performance (cop) for different values of the exponent *m*. Both characteristics, cooling rate and coefficient of performance, reach a maximum and depend strongly on the exponent value. The higher the value of the exponent, the smaller the heat leakage observed.

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