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On the Curzon–Ahlborn efficiency and its connection with the efficiencies of real heat engines

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Abstract

It is acknowledged that the Curzon–Ahlborn efficiency η_{CA} determines the efficiency at maximum power production of heat engines only affected by the irreversibility of finite rate heat transfer (endoreversible engines), but η_{CA} is not the upper bound of the efficiencies of heat engines. This is conceptually different from the role of the Carnot efficiency η_C which is indeed the upper bound of the efficiencies of all heat engines. Some authors have erroneously criticized η_{CA} as if it were the upper bound of the efficiencies of endoreversible heat engines. Although the efficiencies of real heat engines cannot attain the Carnot efficiency, it is possible, and often desirable, for their efficiencies to be larger than their respective maximum power efficiencies. In fact, the maximum power efficiency is the allowable lower bound of the efficiency for a given class of heat engines. These important conclusions may be expounded clearly by the theory of finite time thermodynamics. © 2000 Published by Elsevier Science Ltd.

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1. Introduction

In 1975, Curzon and Ahlborn [1] considered the influence of finite rate heat transfer between the external heat reservoirs and the working fluid on the performance of a Carnot heat engine

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and obtained the efficiency of a Carnot engine at maximum power output as

$$\eta_m = 1 - \sqrt{T_L/T_H} \equiv \eta_{CA}, \tag{1}$$

where T_H and T_L are the temperatures of the hot and cold heat reservoirs, respectively. Although this efficiency was derived independently by Curzon and Ahlborn in 1975, similar systems were considered as early as 1957 by Chambadal and Novikov [2,3], so the efficiency might be called, chronologically, the Chambadal–Novikov–Curzon–Ahlborn efficiency [4]. For the sake of brevity and since it is a well-established name, the efficiency will be simply referred to as the CA efficiency, i.e. η_{CA}.

Curzon and Ahlborn [1] emphasized that "Eq. (1) has the interesting property that it serves as quite an accurate guide to the best observed performance of real heat engines." Because of this statement, some authors have mistaken the CA efficiency as the upper bound on the efficiencies of heat engines. So far, the consequences of the CA efficiency have not been understood very well, and some diverging points of view on the CA efficiency have appeared in journals and textbooks. Thus, it is essential to discuss the meaning of the CA efficiency further. The correct understanding of this issue will be helpful in proper application of finite time thermodynamics to real systems.

2. Efficiency at maximum power output of heat engines with additional irreversibilities or other constraint conditions

The important significance of Eq. (1) is to provide the efficiency at maximum power output of a Carnot heat engine only affected by the irreversibility of finite rate heat transfer. When other irreversibilities or constraint conditions are also considered, in general, the efficiency of a heat engine at maximum power output will be smaller than η_{CA}. Several representative examples are listed below.

When the influence of finite rate heat transfer as well as *finite compression ratio* are taken into account, the efficiency of a Carnot heat engine at maximum power output is reduced to [5]

$$\eta_m = 1 - \sqrt{\frac{T_L}{T_H} - \frac{\left(1 - \sqrt{T_L/T_H}\right)^2}{2} \frac{1/(\gamma - 1)}{\ln r_v}} < \eta_{CA}, \tag{2}$$

where γ is the specific heat ratio, and r_v is the compression ratio.

When the influence of finite rate heat transfer and *irreversibilities inside the working fluid* are taken into account, the efficiency of a Carnot heat engine at maximum power output is given by [6–8]

$$\eta_m = 1 - \sqrt{\frac{IT_L}{T_H}} < \eta_{CA}, \tag{3}$$

where $I > 1$ describes the internal irreversibility occurring in the working fluid.

When the influence of finite rate heat transfer and *heat leak loss* are taken into account, the efficiency of a Carnot heat engine at maximum power output is [9,10]

$$\eta_{\rm m} = \frac{\left(1-\sqrt{T_{\rm L}/T_{\rm H}}\right)^2}{1+(C_i/K)\eta_{\rm C}-\sqrt{T_{\rm L}/T_{\rm H}}} < \eta_{\rm CA}, \tag{4}$$

where C_i is the coefficient of the heat leak loss, K is the equivalent thermal conductance of the heat engine and η_C is the efficiency of a reversible Carnot heat engine.

When the influence of finite rate heat transfer and a *finite heat source* are taken into account, the efficiency of a heat engine at maximum power output is [11]

$$\eta_{\rm m} = 1-\sqrt{T_{\rm L}/T_{\rm H}^*} = 1-\sqrt{T_{\rm L}\frac{C_1\ln\left\{1-\left[\dfrac{Q_1}{(C_1T_{\rm H})}\right]\right\}}{Q_1}} < \eta_{\rm CA}, \tag{5}$$

where C_1 is the heat capacity of the finite heat source, and Q_1 is the heat absorbed from the finite heat source by the working fluid per cycle.

When the influence of heat transfer and *fluid flow irreversibilities* are taken into account, the efficiency of a heat engine at maximum power output is [12]

$$\eta_{\rm m} = \frac{n}{n+1}\left(1-\sqrt{T_{\rm L}/T_{\rm H}}\right) < \eta_{\rm CA}, \tag{6}$$

where n covers the range from laminar flow ($n = 1$) to turbulent flow in the fully rough regime ($n = 2$).

The results quoted above show that when the influence of other irreversibilities or constraint conditions in addition to finite rate heat transfer is considered, the efficiency of a heat engine at maximum power output is smaller than the CA efficiency. However, this does not imply that the CA efficiency is the upper bound on the efficiencies of heat engines. For example, when the Curzon–Ahlborn heat engine is operated under "*ecological*" *optimization* conditions, the efficiency is [13,14]

$$\eta_{\rm E} = 1-\frac{T_{\rm L}}{T_{\rm H}}\frac{\sqrt{1+T_{\rm H}/T_{\rm L}}}{2} > \eta_{\rm CA}, \tag{7}$$

where T_L is equal to the environment temperature. This behavior may be clearly seen from the power output versus efficiency curves of heat engines.

3. Power output versus efficiency of heat engines: which is the optimal operation?

In order to understand the meaning of the CA efficiency, it is necessary to derive the fundamental optimum relation between the power output and the efficiency of a Carnot heat engine. When we only consider the influence of finite rate heat transfer on the performance of a Carnot heat engine, the fundamental optimum relation is given by [15–20]

$$p = \eta\left(1-\frac{1-\eta_{\rm C}}{1-\eta}\right), \tag{8}$$

where p is the dimensionless power output. Using Eq. (8), we can generate the power output versus efficiency curve [6,8,9,21,22] shown in Fig. 1. It is clearly seen from Fig. 1 that when the power output of a Carnot heat engine attains the maximum, the efficiency is equal to the CA efficiency. When $p < p_{max}$, there are two different efficiencies for a given power output, where one is smaller than the CA efficiency and the other is larger than the CA efficiency. When the efficiency lies in the region of $\eta < \eta_{CA}$, the efficiency decreases as the power output decreases. Obviously, the region of $\eta < \eta_{CA}$ is not desirable. When the efficiency lies in the region of $\eta > \eta_{CA}$, the efficiency increases as the power output decreases, and vice versa. Thus, the optimal operation of an endoreversible Carnot heat engine should be the region of $\eta_C > \eta \geq \eta_{CA}$ [15,19,22]. The "ecological" optimization condition mentioned above is just in this region. Fig. 1 also shows clearly the fact that when the efficiency of a heat engine is larger than

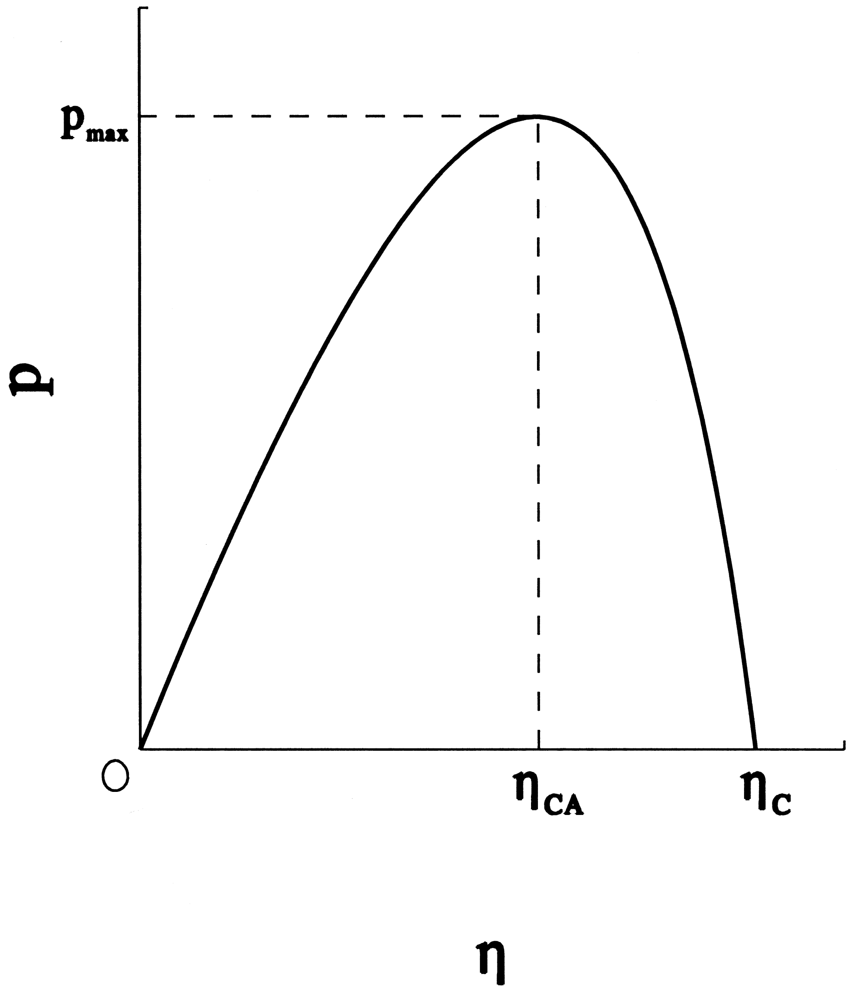

Fig. 1. The dimensionless power output versus efficiency curve of an endoreversile Carnot heat engine affected by finite rate heat transfer.

η_{CA}, its power output is not the maximum; and the larger the efficiency is, the smaller the power output is. This is one important conclusion of finite time thermodynamics.

In a recent article [23], Gyftopoulos pointed out that "the good correlation between the thermal efficiencies of the old power plants and Eq. (1) must be a numerical accident for at least two reasons. First, the plants that are considered in the comparisons were not designed to achieve the optimum thermal efficiency of finite time thermodynamics and, therefore, it would be a miracle if each self-optimized itself to the point of best performance. Second, and perhaps more importantly, last year General Electric announced the great achievement of a gas-turbine, combined-cycle power plant with a thermal efficiency of 60%." Gyftopoulos' statement, which has been proliferated in another article [24], just exemplifies that the efficiencies of real heat engines designed optimally may be definitely larger than the CA efficiency. This is obviously correct because it is an observed experimental fact. However, it is worthwhile to note that this experimental "discovery" is in full agreement with the important conclusion of finite time thermodynamics mentioned above because the CA efficiency is *not* the upper bound on the efficiency.

Although Gyftopoulos has noted this experimental observation, it is a pity that he has not interpreted it correctly. He stated that "if this efficiency (i.e., 60%) correlated with the results of finite time thermodynamics, it would imply a ratio $T_L/T_H = 0.16$ and, therefore, an available energy or exergy of the products of combustion of the gas of about 84% of the exergy of natural gas! As a result, we would be compelled to conclude that the loss of exergy of the fuel upon combustion is only 16% though every calculation and every measurement yield a loss of about 30%." In this statement [23,24], Gyftopoulos ascribes the lack of agreement between the measurements and the CA efficiency to basic shortcomings of finite time thermodynamics. This sweeping conclusion is not well founded, because an irrational assumption was hidden in the statement.

A closer look at Gyftopoulos' claim reveals that he has assumed implicitly that the power plant with a thermal efficiency of 60% is operated at the state of maximum power output. Following Gyftopoulos' point of view, if other irreversibilities besides finite rate heat transfer were considered in the model, the difference between the theoretical result and the measurement would become larger because the maximum power efficiency of a heat engine affected by finite rate heat transfer and other additional irreversibilities is, in general, smaller than η_{CA} (Section 2).

When interpreting such discrepancies between experimental findings and model calculations, one must always (i) check that the real operating conditions and the modeling objectives are consistent and (ii) consider the aim of the model (e.g. high accuracy, transparency, or a bound). Neither was done here. The CA objective was maximum power production, whereas real power plants have other objectives, usually economic ones [25]. Further, finite time thermodynamic models are not explicitly designed for maximum agreement with experimental detail — that is the realm of engineering design — but rather for physical insight and easy interpretation. Thus, it is evident that Gyftopoulos' claim is incorrect. In general, real heat engines, particularly heat engines with high efficiencies, are not operated at the state of maximum power output. One must not take the CA efficiency as the upper bound on the efficiency, much less denigrate finite time thermodynamics, in general, based on this misinterpretation.

As mentioned above, the result of finite time thermodynamics shows that the optimal efficiency of endoreversible Carnot heat engines should be $\eta \geq \eta_{CA}$. It is, thus, clear that it is allowable for an optimally designed power plant not to be operated in the state of maximum power output. If the power plant is operated at a certain state of $\eta > \eta_{CA}$, say η_E, it is irrational to use the maximum power criterion to analyze the performance of the power plant and to derive other conclusions. If one wants to understand the performance of a heat engine, it is necessary to start from the fundamental optimum relation of the heat engine. Consequently, finding the fundamental optimum relation of a heat engine is the kernel of finite time thermodynamic cycle theory and is more important than finding e.g. the maximum power efficiency of the heat engine, since the latter can be derived directly from the former. In addition, starting from the fundamental optimum relation, one can easily determine the optimum operating region and optimize other performance parameters of a heat engine besides the maximum power efficiency.

In another journal [26], Sekulic stated that "a finite time thermodynamics figure of merit (i.e., the maximum power efficiency η_{FTT}) should always be smaller than the Carnot efficiency. Consequently, the margin for improvement of a real heat engine, calculated on the basis of Eq. (1), should always be smaller than the corresponding margin implied by the Carnot efficiency. In other words, the following sequence of thermodynamic efficiency margin definitions and inequalities should hold: $\Delta\eta_{Carnot} = \eta_{Carnot} - \eta_{Actual} > \Delta\eta_{FTT} = \eta_{FTT} - \eta_{Actual} > 0$. Thus, the margin for improvement of a real system based on a finite time thermodynamics figure of merit, $\Delta\eta_{FTT}$, intends to provide a more realistic assessment of a maximum achievable improvement in comparison to the margin $\Delta\eta_{Carnot}$ calculated using Carnot efficiency. Also note that the following inequality must be satisfied in all situations $\Delta\eta_{FTT} \neq 0$." In this last statement, Sekulic, like Gyftopoulos above, has mistaken the CA efficiency to be the maximum efficiency of irreversible heat engines. The inequality $\Delta\eta_{FTT} = \eta_{FTT} - \eta_{Actual} > 0$ violates the theory of finite time thermodynamics and is not true.

Early in 1988, Bejan [27] pointed out that the CA efficiency is not the "maximum" efficiency of a heat engine. In other words, it should not be confused with the Carnot efficiency. In some earlier literature [15], Yan pointed out that the CA efficiency is not the maximum efficiency of a heat engine but determines the lower bound on the optimal efficiency of a heat engine affected by finite rate heat transfer. This correct point of view is also advanced in de Vos' book [19]. It is, thus, obvious that although the efficiencies of real heat engines cannot attain the Carnot efficiency, it is possible, and is often desirable, for the efficiencies to exceed the respective maximum power efficiencies.

4. Optimum operating criteria

When different irreversibilities are taken into account, the maximum power efficiency η_m of a heat engine will be different from the CA efficiency. However, both η_m and η_{CA} have similar significance. They both determine the lower bound for the optimal efficiencies of their respective class of heat engines. Thus,

$$\eta \geq \eta_m \tag{9}$$

is an optimality criterion for heat engines with the specified irreversibilities. If the efficiency of a heat engine, adequately described by the choice of irreversibilities, does not satisfy Eq. (9), the heat engine is not designed or operated optimally.

When just finite rate heat transfer and heat leak loss are considered, the fundamental optimum relation of a heat engine may be used to generate the power output versus efficiency curve [9,10,21,22] of the heat engine shown in Fig. 2. In this case, the power output versus efficiency curve is of a loop shape, which is very different from that of a Carnot heat engine affected only by finite rate heat transfer. There exist a maximum power output and a maximum efficiency with non-zero power output. The optimal operating region of this heat engine is situated on that part of the power output versus efficiency curve which has a negative slope

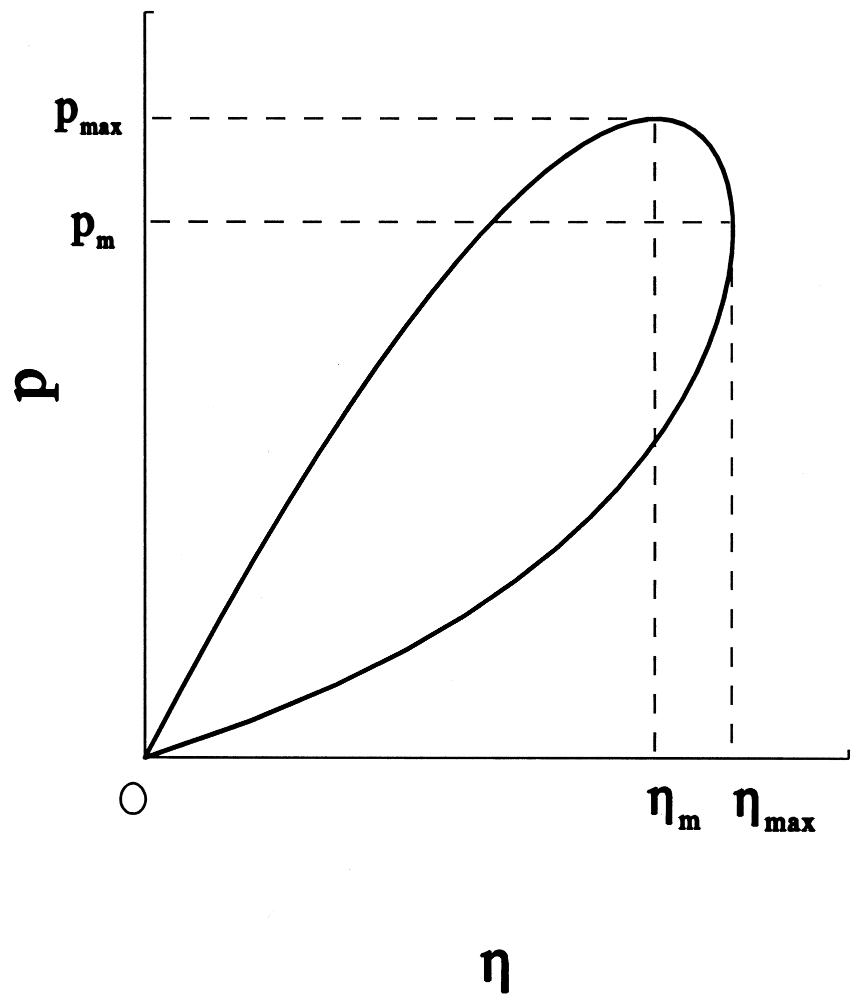

Fig. 2. The dimensionless power output versus efficiency curve of a Carnot heat engine affected by finite rate heat transfer and heat leak loss. $p_{\max}$ is the dimensionless maximum power output and p_{m} is the dimensionless power output at maximum efficiency $\eta_{\max}$.

[9,10,22]. Thus, besides Eq. (9), the dimensionless power output of a heat engine must be constrained by

$$p \geq p_{\mathrm{m}}, \tag{10}$$

where p_{m} is the dimensionless power output at maximum efficiency.

It is important to note all the information emerging from Fig. 2. When the cycle time is equal to two particular finite values, the power output and the efficiency will attain their maximum values, respectively. When the cycle time tends to zero or infinity, both the power output and the efficiency of the cycle approach zero. This shows that when certain irreversibilities are considered, an infinite time process may be irreversible. Gyftopoulos et al. provided an experimental example of this [23,28]. Having "two identical electricity storage batteries with an internal discharge time constant of 100 days, we can discharge one battery very slowly, say over 10^4 days, and the other very fast, say over 1 day, and ask 'Which discharge process is closer to reversible?' As is very well known from billions of experiments, the fast process is very close to reversible, whereas the slow process is totally irreversible, because in the fast process practically all the stored adiabatic availability is transferred out, whereas in the slow process practically no adiabatic availability is transferred; it is all dissipated in the battery." Such an experiment is not contrary to the theory of finite time thermodynamics. According to finite time thermodynamic analysis [9], when the thermal resistance (equivalent to the internal resistance of a battery) is dominant for a power plant, a slow process is closer to reversible; but when the heat leak loss (corresponding to the internal discharge of the battery) is dominant, a fast process is closer to reversible. Thus, with internal irreversibilities, a fast discharge process may be closer to reversible than a slow one. Quite a similar problem, the maximum work from an electric battery model with an internal discharge, has been analyzed theoretically [29]. Thermal systems with internal structure were also analyzed a long time ago [30].

5. Conclusions

Finite time thermodynamics is a science. It is an extension of classical thermodynamics in which the unavoidable minimum irreversible losses are considered for the purpose of gaining insight. Such a new branch of learning is rich in content, not just the maximum power efficiency of a simple Curzon–Ahlborn heat engine or endoreversible cycles. Over the last two decades, finite time thermodynamics has been successfully used in many regions in physics and other subjects. It is necessary to further develop and generalize the theory of finite time thermodynamics.

In order to understand the performance of an endoreversible or irreversible heat engine, one must find the fundamental optimum relation of the heat engine. Only in this way can one realize the genuine meaning of the maximum power efficiency. An irreversible process may take part in a finite or infinite interval in time. The performance of such an irreversible process may be expounded very well by using the theory of finite time thermodynamics.

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